Wave-Particle duality and Electron diffraction

Louis de-Broglie proposed that all objects with total momentum p will have an 'associated wavelength.' If this wavelength is on a similar scale to an *aperture*, then *significant diffraction effects should be observed*. Although the electron had been established as a *particle*, i.e. an object localized in space with a defined mass and velocity, Germer and Davisson showed that 'electron waves' can be diffracted by the atomic lattice associated with a disk of carbon atoms. *Wave-particle duality* is certainly measurable for electrons i.e.*both models* are appropriate descriptions of its physical characteristics.

$$e = 1.6021766208(98)\times 10^{-19}\,\mathrm{C}$$
$$c = 2.99792458\times 10^{8}\,\mathrm{ms^{-1}}$$
$$h = 6.626070040(81)\times 10^{-34}\,\mathrm{kgm^2s^{-1}}$$
$$m_e = 9.10938356(11)\times 10^{-31}\,\mathrm{kg}$$

Louis de Broglie
1892 – 1987
Nobel Prize 1929

The de Broglie formula expressing the relationship between wavelength and momentum can be 'derived' by comparing Einstein's famous mass-energy relationship with the classical expression for the momentum of a particle travelling at the speed of light. Although the argument is flawed, i.e. Special Relativity shows that *only massless particles* can travel at the speed of light, it does produce the correct result! The de Broglie analysis also alludes to another truth, that although photons are indeed *massless,* they do possess *momentum*

$E = mc^2$ Energy

$p = mc$ Momentum

$\therefore E = pc$

$E = hf$ Planck formula

$c = f\lambda$ Wave velocity

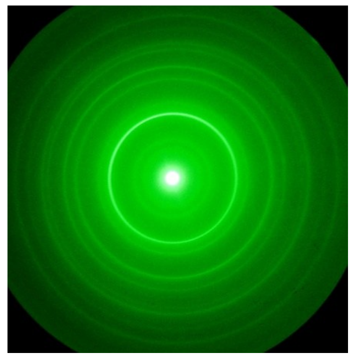
$\therefore E = \dfrac{hc}{\lambda}$ $\therefore pc = \dfrac{hc}{\lambda}$

$\therefore \boxed{\lambda = \dfrac{h}{p}}$ **de Broglie relation**

Classical calculation for electron wavelength resulting from electron accelerated by voltage V

$$eV = \tfrac{1}{2}m_e v^2,\ \ p = m_e v$$
$$\therefore eV = \tfrac{1}{2}m_e\left(\frac{p}{m_e}\right)^2 = \frac{p^2}{2m_e}$$
$$p = \sqrt{2m_e eV}$$
$$\lambda = \frac{h}{p}$$
$$\boxed{\lambda = \frac{h}{\sqrt{2m_e eV}}}$$

Electron wave diffraction rings



Relativistic calculation for electron wavelength resulting from electron accelerated by voltage V

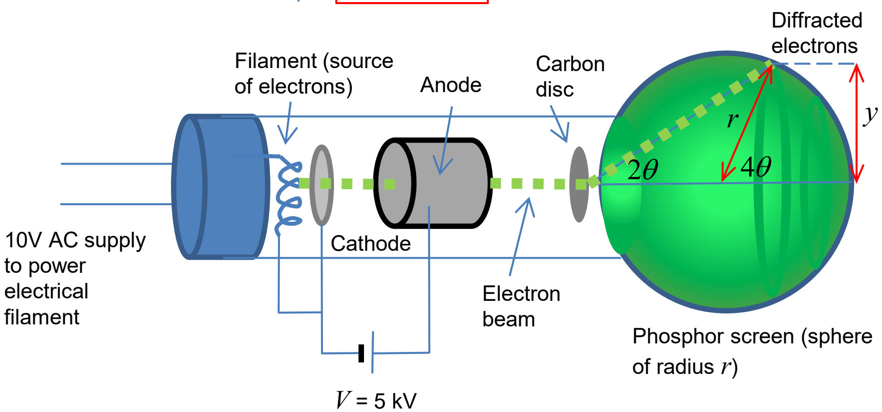
$E = m_e c^2 + eV$ Total energy including rest mass

$E^2 = p^2c^2 + m_e^2c^4$ Energy-momentum invariant

$$\left(m_e c^2 + eV\right)^2 = p^2c^2 + m_e^2c^4$$
$$m_e^2c^4 + 2m_e c^2 eV + e^2V^2 = p^2c^2 + m_e^2c^4$$
$$p^2 = \frac{2m_e c^2 eV + e^2V^2}{c^2}$$
$$p = \sqrt{2m_e eV + \frac{e^2V^2}{c^2}} = \sqrt{2m_e eV}\sqrt{1 + \frac{eV}{2m_e c^2}}$$
$$\lambda = \frac{h}{p}$$
$$\boxed{\lambda = \frac{h}{\sqrt{2m_e eV}}\frac{1}{\sqrt{1 + \frac{eV}{2m_e c^2}}}}$$

Lester Germer (right) with Clinton Davisson in 1927.
Nobel Prize 1937.

Voltage = 3kV
Classical electron wavelength = 0.022391nm
Relativistic Factor (RF) = 0.99854
Electron wavelength = 0.022359nm
Diffraction ring 1 : 4*theta = 12 degrees, y = 1.3cm
Diffraction ring 2 : 4*theta = 24.1 degrees, y = 2.55cm
Diffraction ring 3 : 4*theta = 36.2 degrees, y = 3.69cm
Diffraction ring 4 : 4*theta = 48.5 degrees, y = 4.68cm
Diffraction ring 5 : 4*theta = 60.9 degrees, y = 5.46cm



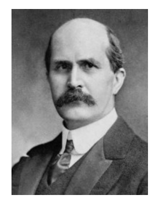
d, the atomic spacing of carbon atoms can be found from the gradient of $\tfrac{1}{2}n\lambda$ plotted against $\sin\theta$.

$$y = r\sin 4\theta\quad \therefore \theta = \tfrac{1}{4}\sin^{-1}\left(\frac{y}{r}\right)$$
$$\tfrac{1}{2}n\lambda = d\sin\theta$$

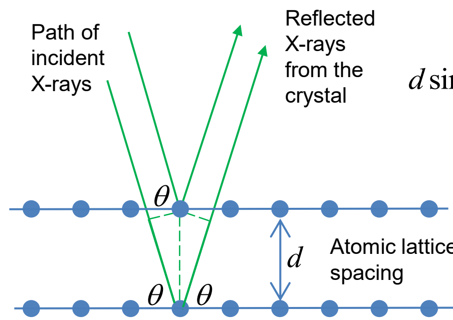
$$\lambda = \frac{h}{\sqrt{2m_e eV}}$$

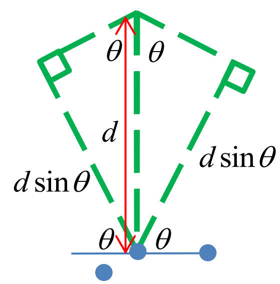
Bragg's law of X-ray diffraction from atoms in a crystal lattice

$$\boxed{2d\sin\theta = n\lambda}$$

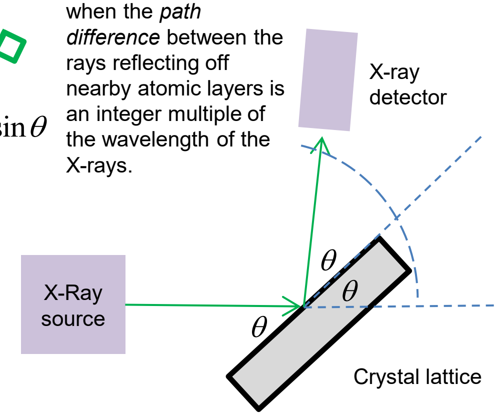


Sir William Henry Bragg 1862-1942





Reflected rays will constructively interfere when the *path difference* between the rays reflecting off nearby atomic layers is an integer multiple of the wavelength of the X-rays.



*Perhaps the de Broglie relation should be taken as *axiomatic*