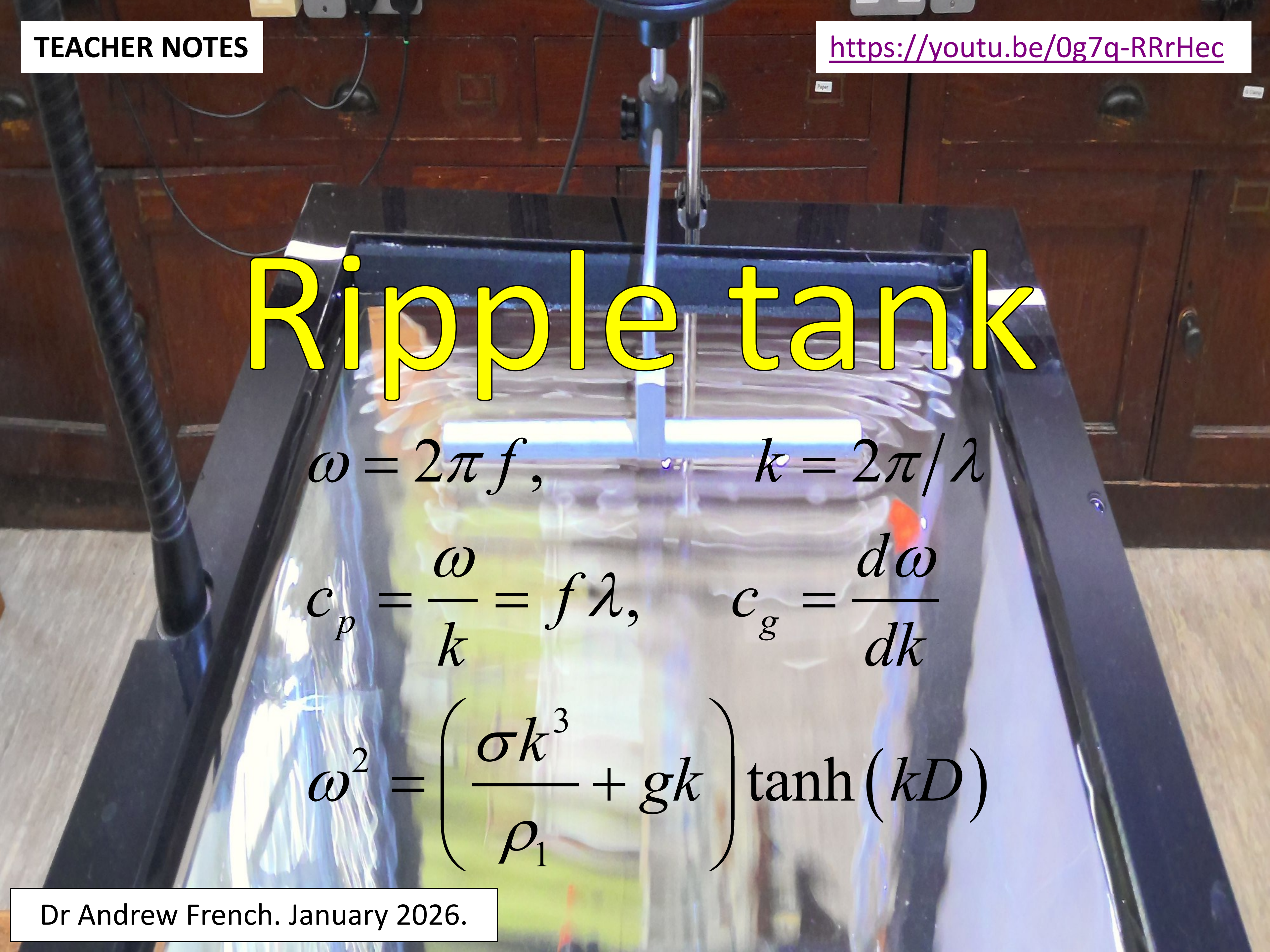
TEACHER NOTES

https://youtu.be/0g7q-RRrHec

Ripple tank



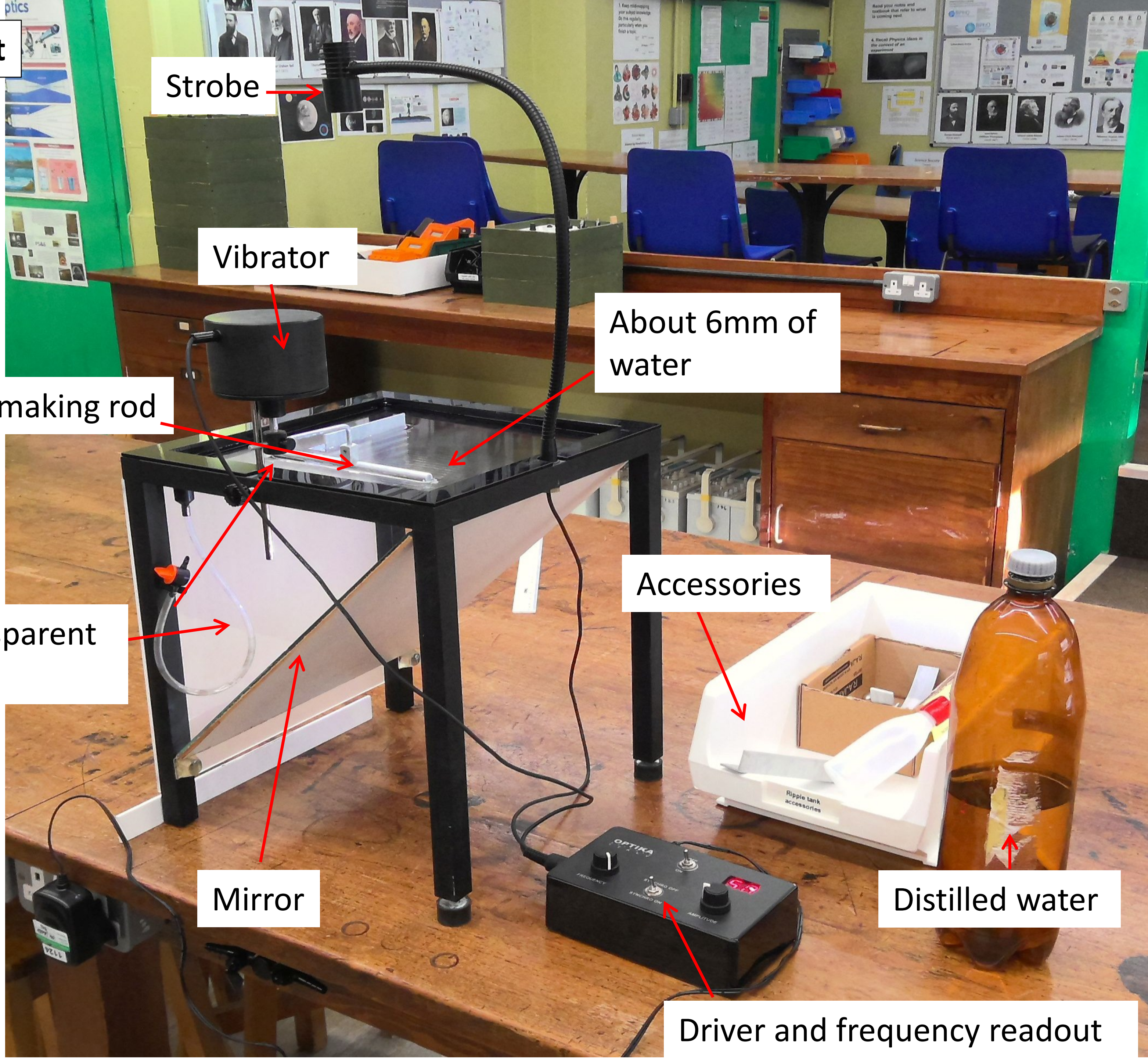
$$\omega = 2\pi f, \qquad k = 2\pi / \lambda$$

$$c_p = \frac{\omega}{k} = f\lambda, \qquad c_g = \frac{d\omega}{dk}$$

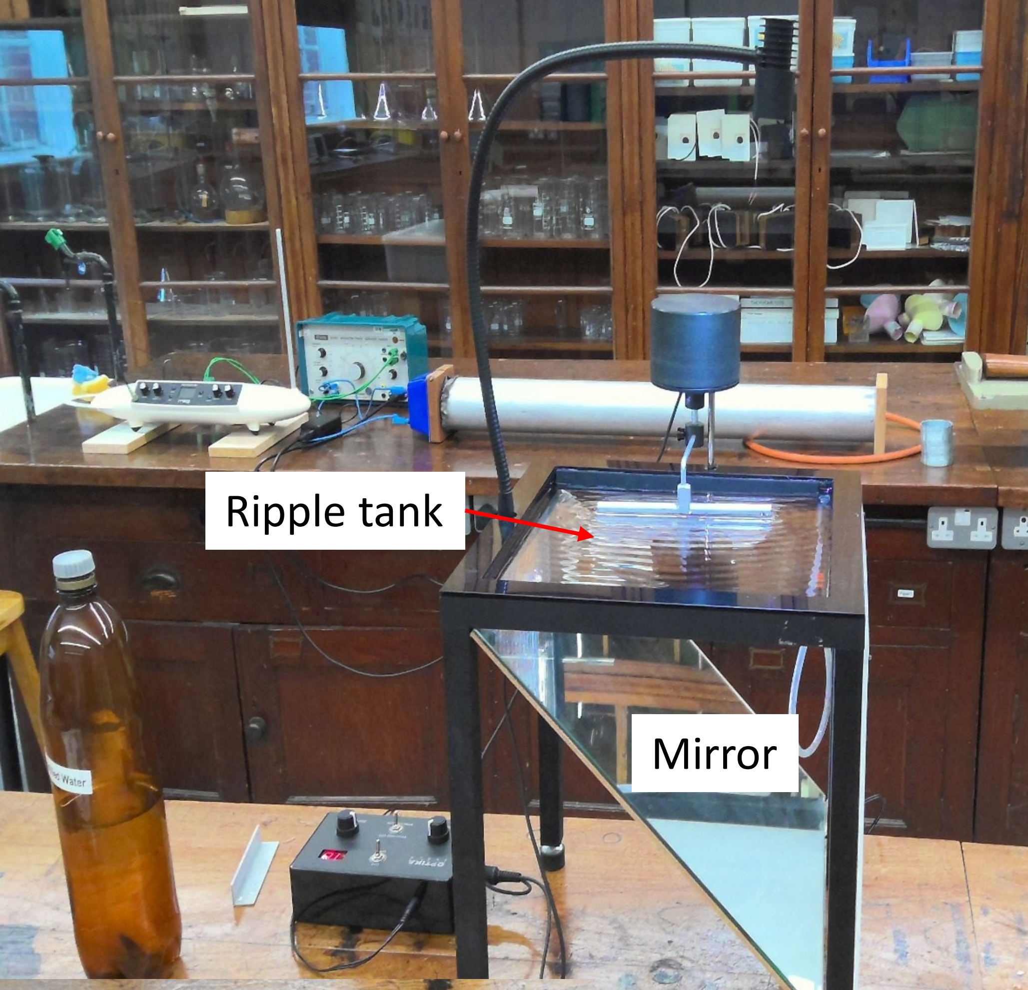
$$\omega^2 = \left(\frac{\sigma k^3}{\rho_1} + gk \right) \tanh(kD)$$

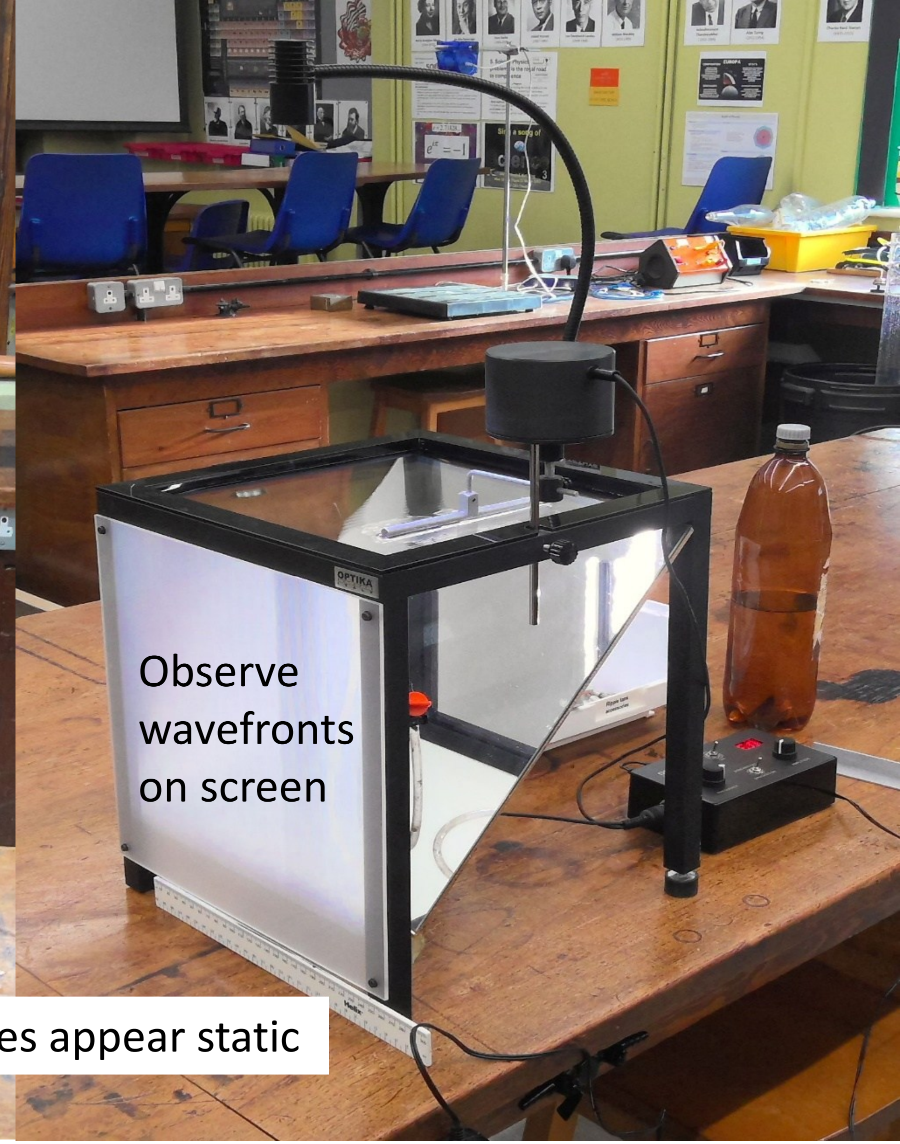
Dr Andrew French. January 2026.

Equipment



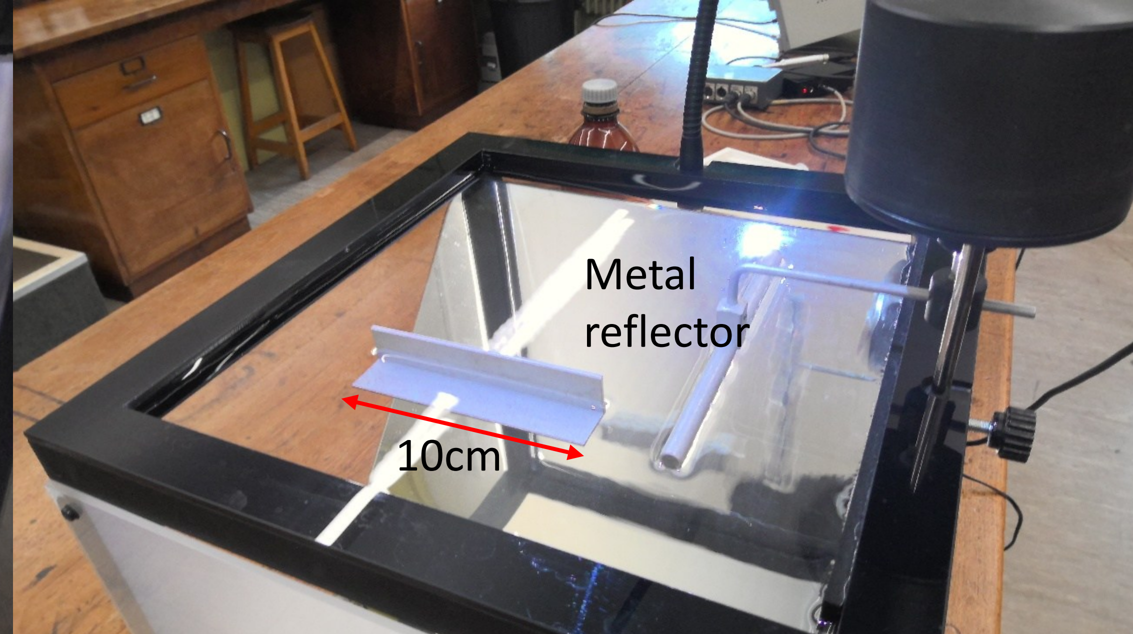
Strobe
Vibrator
About 6mm of water
Wave-making rod
Semi-transparent screen
Accessories
Mirror
Distilled water
Driver and frequency readout

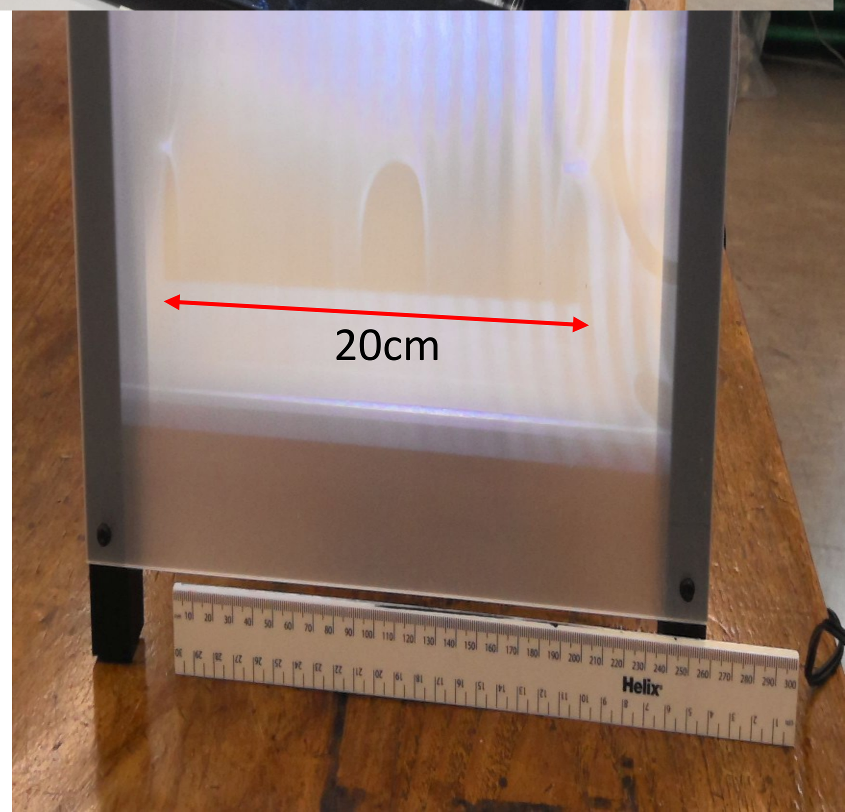




Synchro on sets strobe to wave frequency, so waves appear static

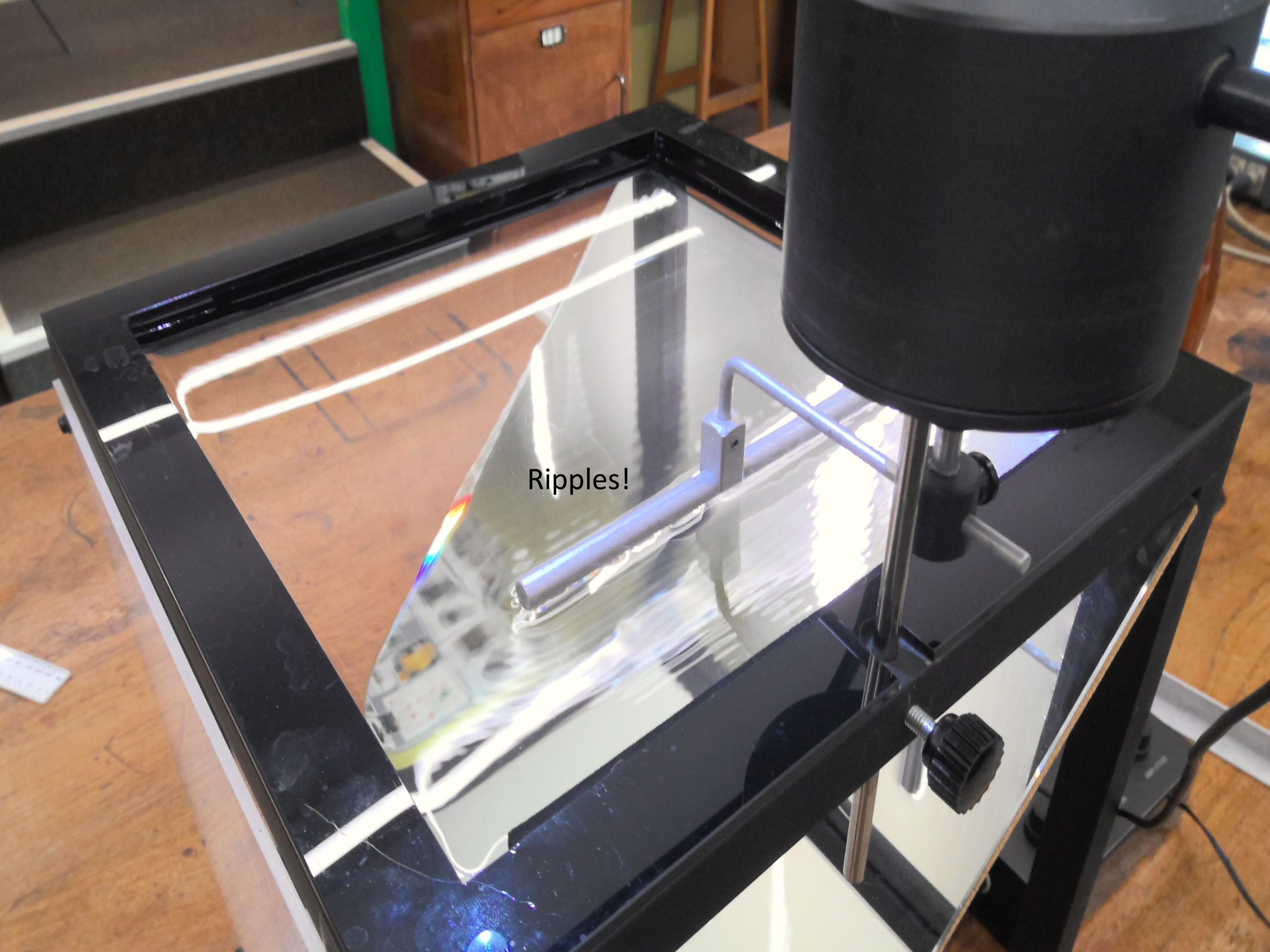
Wave driver control (main powered).
Range of frequencies is 0 to 50Hz. You will probably struggle to make meaningful wavelength measurements below about 13 Hz.





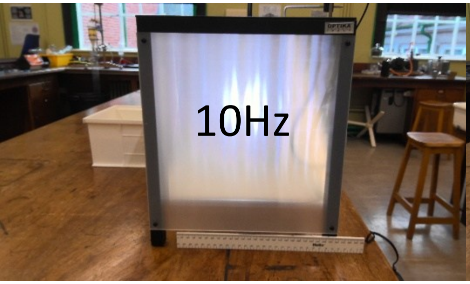
Measuring off the semi-transparent screen is a **magnification factor of x 2.00.**

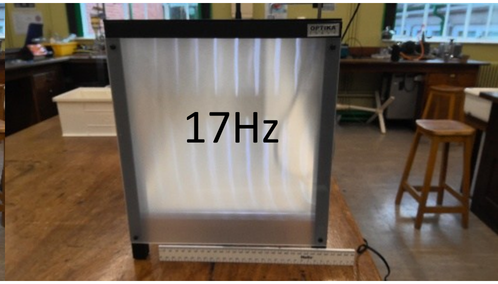
You can see this by placing one of the (10cm long) accessories in the tank and measuring the length of the shadow (20cm).

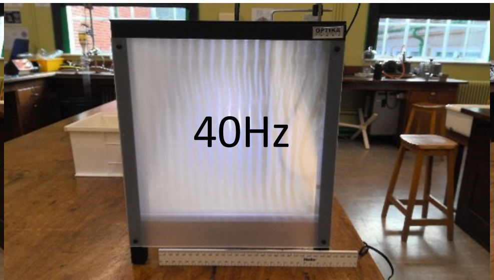

Ripples!

The photographs below illustrate varying the ripple tank frequency from 5Hz to 50Hz.

I used a water depth of 6mm. You can easily make direct wavelength measurements from the screen using a ruler. **Just remember to halve the measurements due to the magnification!**







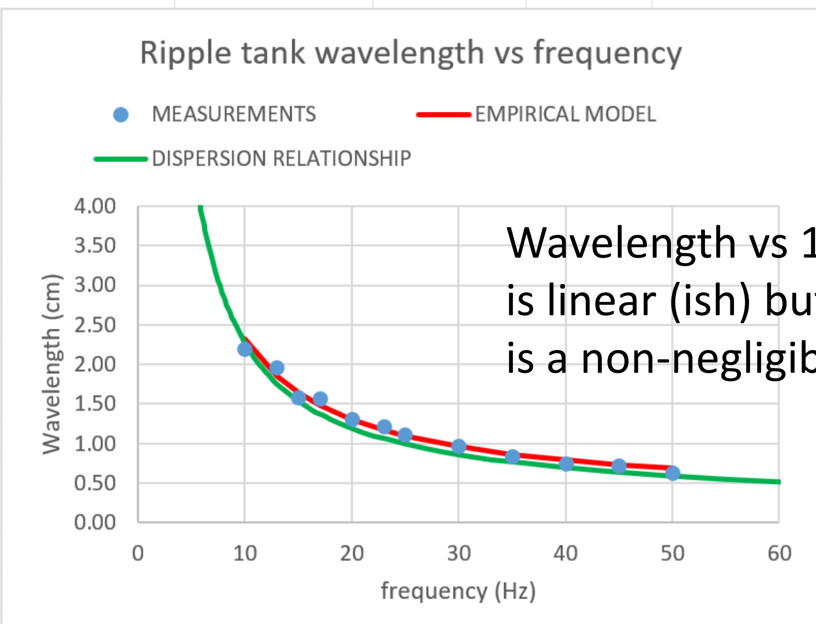
from strobe box		measurements from photo					
frequency (Hz)	1/f (Hz^-1)	Pixels for N waves	Pixels for 30 cm ruler	Number of wavelengths N	wavelength /cm	phase velocity c = f*lamda (cm/s)	MODEL wavelength (cm)
10	0.100				2.20	22.0	2.32
13	0.077	1066	1628	5	1.96	25.5	1.85
15	0.067	841	1594	5	1.58	23.7	1.64
17	0.059	1149	1572	7	1.57	26.6	1.48
20	0.050	1042	1494	8	1.31	26.2	1.30
23	0.043	1198	1640	9	1.22	28.0	1.17
25	0.040	893	1509	8	1.11	27.7	1.10
30	0.033	887	1523	9	0.97	29.1	0.96
35	0.029	853	1533	10	0.83	29.2	0.86
40	0.025	800	1594	10	0.75	30.1	0.79
45	0.022	885	1541	12	0.72	32.3	0.73
50	0.020	635	1497	10	0.64	31.8	0.69

$$c_p = f\lambda$$

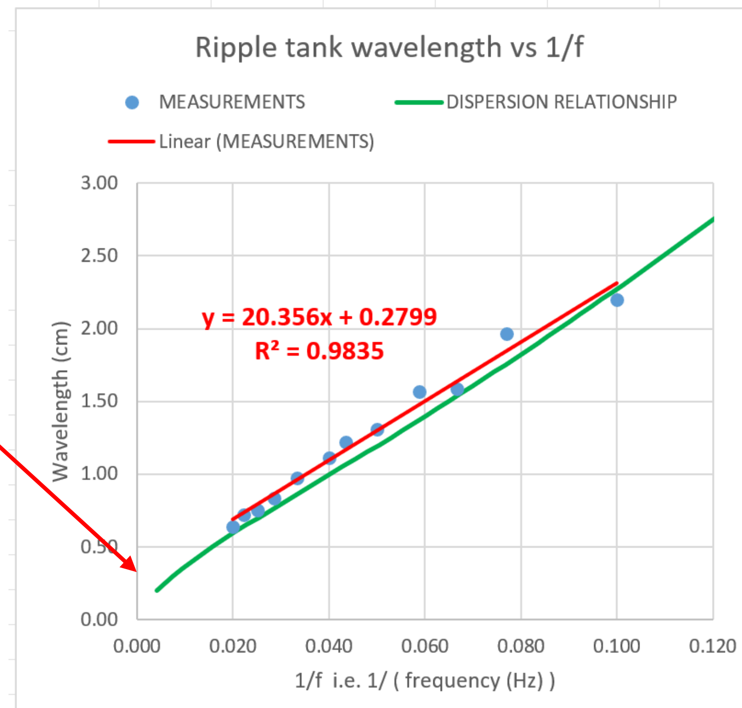
Observe that the *phase velocity* of ripples appears to increase from about 22 cm/s to 32 cm/s between 10Hz and 50Hz.

Empirical Model

$$\lambda/\text{cm} \approx \frac{20.4}{(f/\text{Hz})} + 0.28$$



Wavelength vs 1/f
is linear (ish) but there
is a non-negligible intercept.



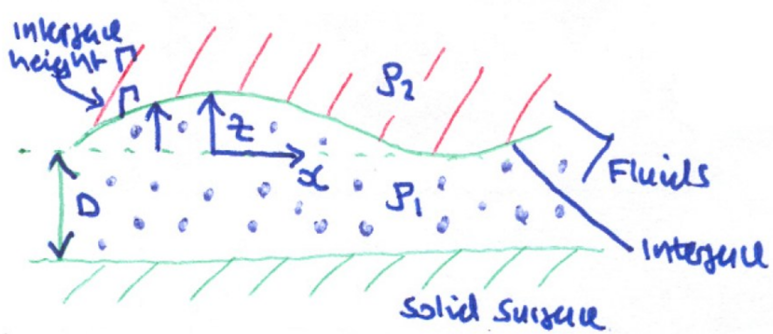
Surface tension of water [edit]

The surface tension of pure liquid water in contact with its vapor has been given by IAPWS[40] as

$$\sigma = 235.8\left(1-\frac{T}{T_C}\right)^{1.256}\left[1-0.625\left(1-\frac{T}{T_C}\right)\right]\ \mathrm{mN/m},$$

where both T and the critical temperature T_C = 647.096 K are expressed in kelvins. The region of validity the entire vapor–liquid saturation curve, from the triple point (0.01 °C) to the critical point. It also provides reasonable results when extrapolated to metastable (supercooled) conditions, down to at least −25 °C. This formulation was originally adopted by IAPWS in 1976 and was adjusted in 1994 to conform to the International Temperature Scale of 1990.

The uncertainty of this formulation is given over the full range of temperature by IAPWS.[40] For temperatures below 100 °C, the uncertainty is ±0.5%.



Water surface tension (N/m) at 25 deg C
0.0731

Temperature /K
291

strength of gravity (N/kg)(
9.8

DISPERSION RELATIONSHIP

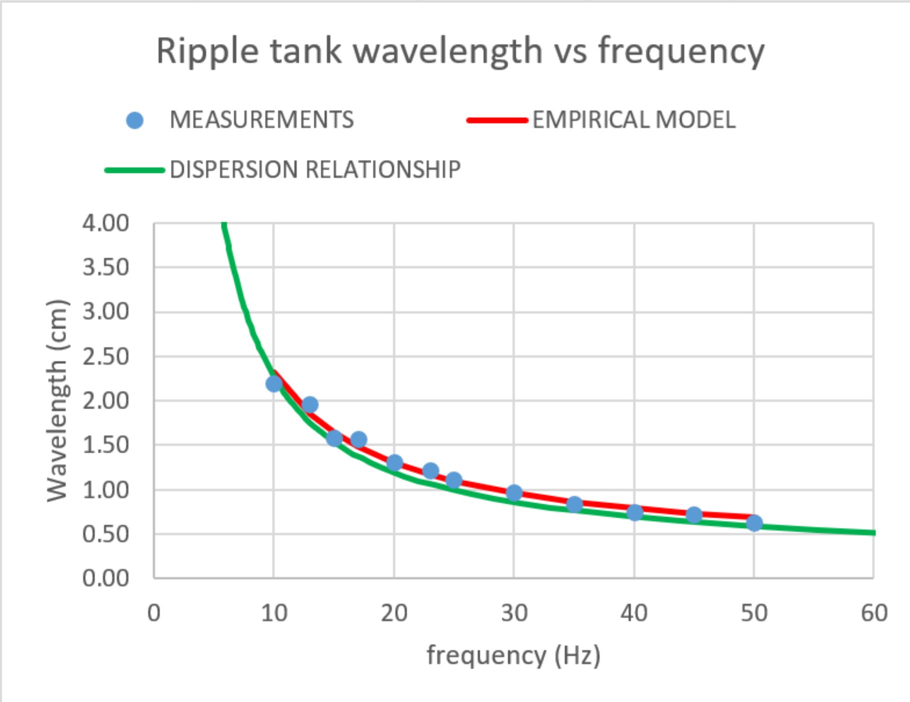
water density (kg/m^3)
1000

Dispersion relationship

$$\omega^2 = \left(\frac{\sigma k^3}{\rho_1} + gk\right)\tanh(kD)$$

lamda (cm)	k (m^-1)	w^2	f (Hz)	1/f	phase velocity w/k (cm/s)
0.2	3.142E+03	2.296E+06	241.17	0.004	48.23
0.25	2.513E+03	1.184E+06	173.21	0.006	43.30
0.3	2.094E+03	6.917E+05	132.37	0.008	39.71
0.35	1.795E+03	4.403E+05	105.60	0.009	36.96
0.4	1.571E+03	2.986E+05	86.96	0.011	34.79
0.45	1.396E+03	2.126E+05	73.38	0.014	33.02
0.5	1.257E+03	1.573E+05	63.12	0.016	31.56
0.55	1.142E+03	1.201E+05	55.16	0.018	30.34

https://en.wikipedia.org/wiki/Surface_tension



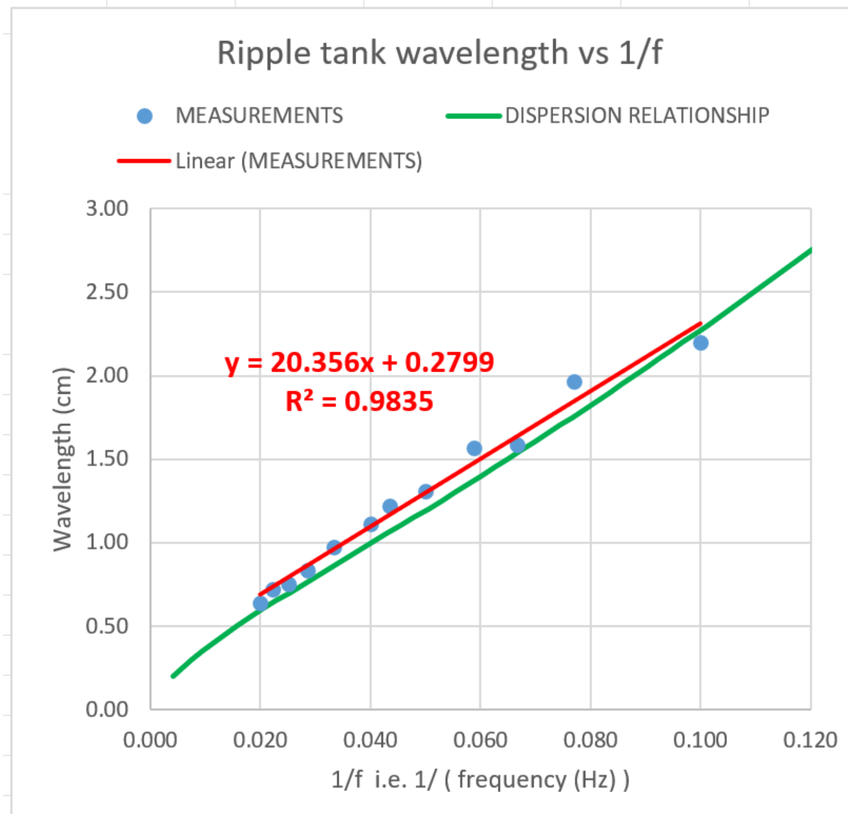
The dispersion relationship (green) is plotted against the measurements and an empirical model fit.

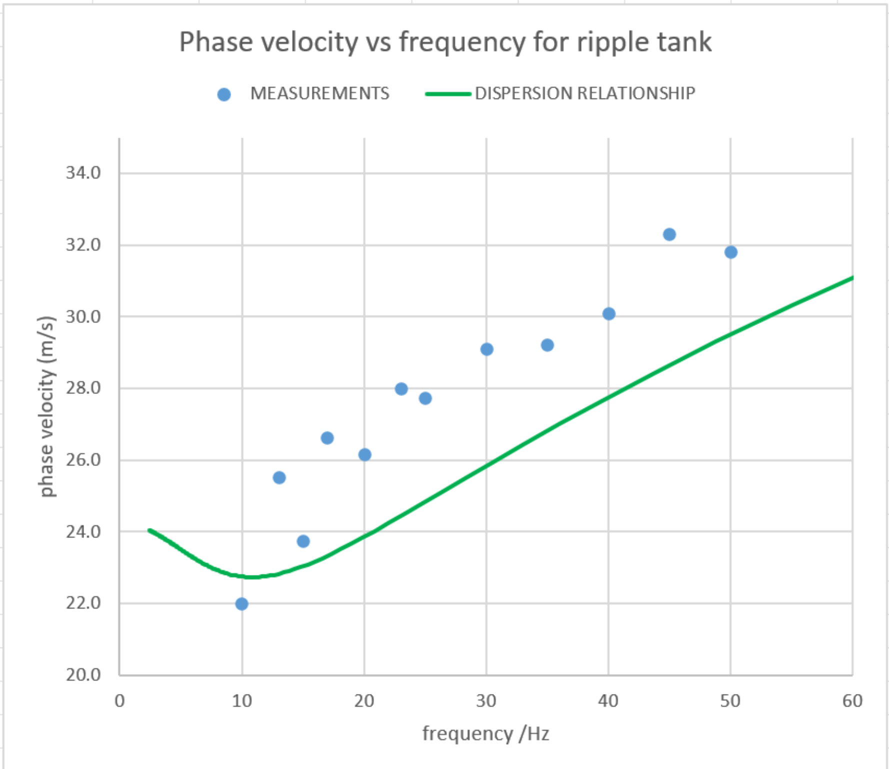
There is qualitative agreement (note error bars have been omitted in this initial analysis).

$$\omega = 2\pi f, \qquad k = 2\pi/\lambda$$

$$c_p = \frac{\omega}{k} = f\lambda, \qquad c_g = \frac{d\omega}{dk}$$

$$\omega^2 = \left(\frac{\sigma k^3}{\rho_1} + gk\right)\tanh(kD)$$





$$\omega = 2\pi f, \qquad k = 2\pi/\lambda$$

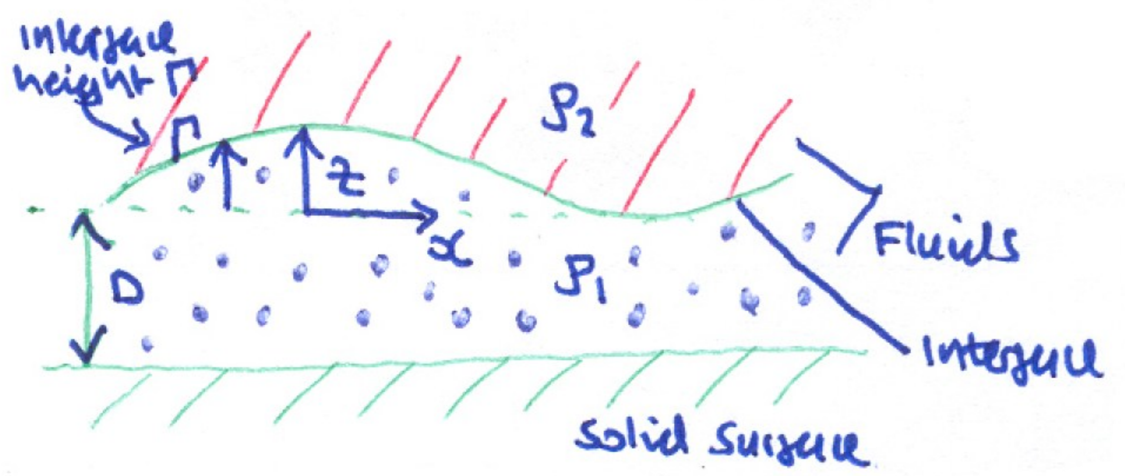
$$c_p = \frac{\omega}{k} = f\lambda, \qquad c_g = \frac{d\omega}{dk}$$

$$\omega^2 = \left(\frac{\sigma k^3}{\rho_1} + gk \right) \tanh(kD)$$

Phase velocity (measured) vs predictions from dispersion relationship is less well correlated, although perhaps follows the general upwards trend with the right gradient.
Note there is a *minimum phase velocity for ripples.*

The measured phase velocity appears to be an overestimate, which is perhaps consistent with distortions caused by taking measurements from photographs.

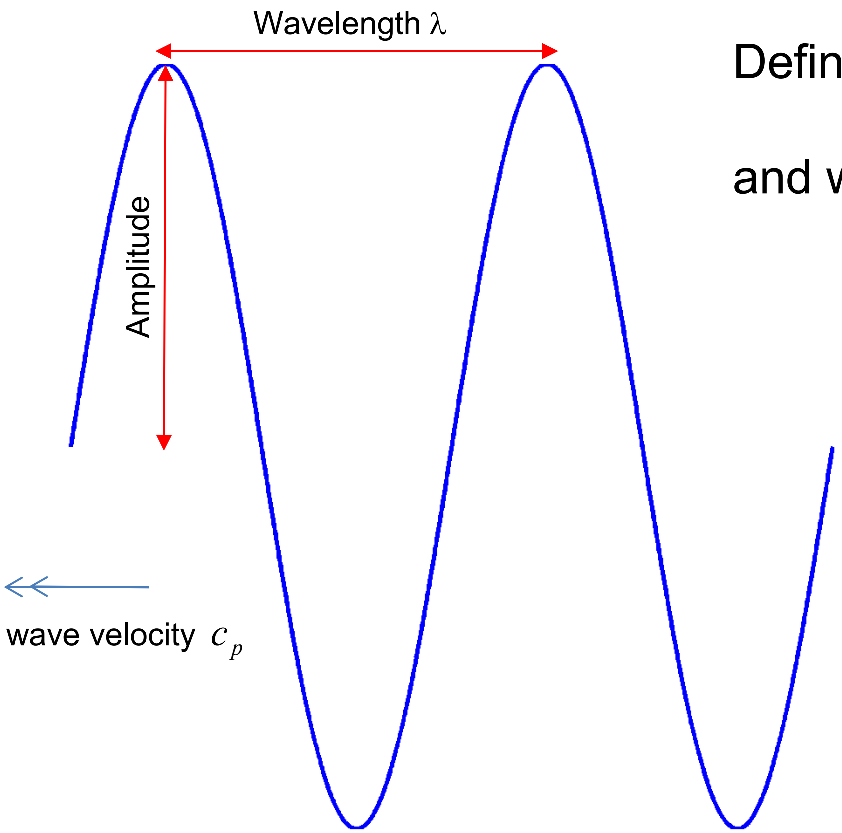
It would be interesting to see a repeat of the experiment using only direct ruler-based measurements off the screen.



Mathematical theory of surface waves

Key topics:
Frequency
Wavenumber
Dispersion relationship
Phase and group velocity
Shallow and deep water waves
Minimum velocity of deep water ripples

A means of characterizing a wave-like disturbance is via a *dispersion relation*. This is an equation which relates the frequency of the wave to its wavelength, plus other parameters such as surface tension, fluid density, depth etc.



Define angular frequency $\omega = 2\pi f$

and wavenumber $k = \dfrac{2\pi}{\lambda}$

For deep water gravity waves, the dispersion relationship is

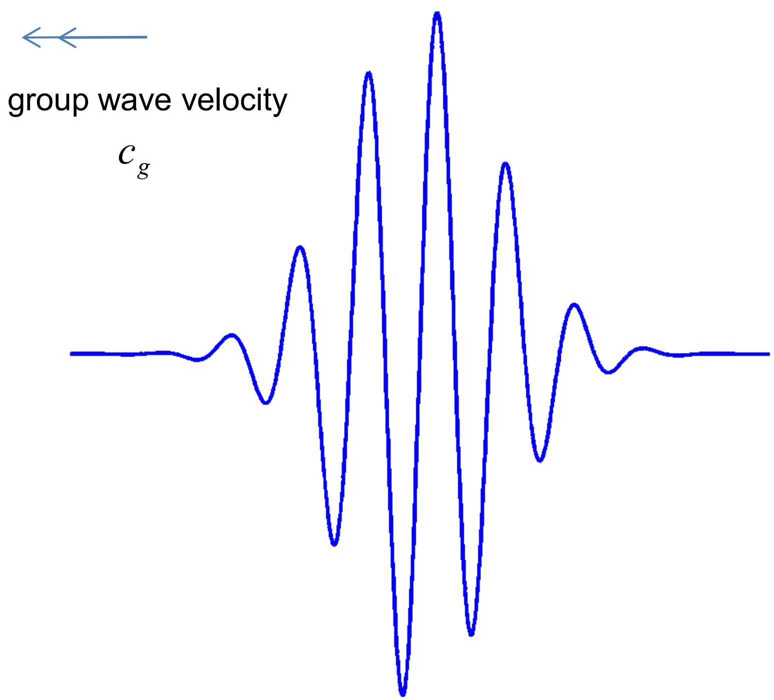
$$\omega^2 = gk$$

i.e. we ignore effects of the density of air, surface tension and assume the depth $D >> \lambda$ i.e. $kD >> 1$

The *phase velocity* (of individual wave crests) can be found from the dispersion relationship

$$c_p = f\lambda = \frac{\omega}{k}$$

The dispersion relationship allows to compute how fast groups of disturbances will travel. This *group velocity* can be *different* from the phase velocity. Relative motion of the wave crests to the overall envelope causes dispersion.



$$c_g = \frac{d\omega}{dk}$$

For deep water gravity driven waves

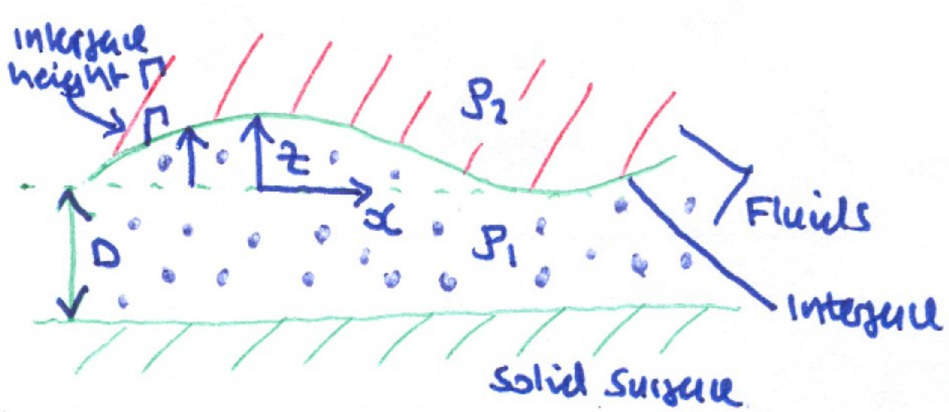
$$\omega = \sqrt{g}k^{\frac{1}{2}}$$

$$c_p = \frac{\omega}{k} = \sqrt{g}k^{-\frac{1}{2}}$$

$$c_g = \frac{d\omega}{dk} = \frac{1}{2}\sqrt{g}k^{-\frac{1}{2}}$$

$$\therefore \quad c_g = \frac{1}{2}c_p$$

In general, for *vorticity* free waves on the interface of two *incompressible, Newtonian* fluids, the dispersion relationship can be shown to be, for waves with amplitude $\Gamma \ll D$



surface tension

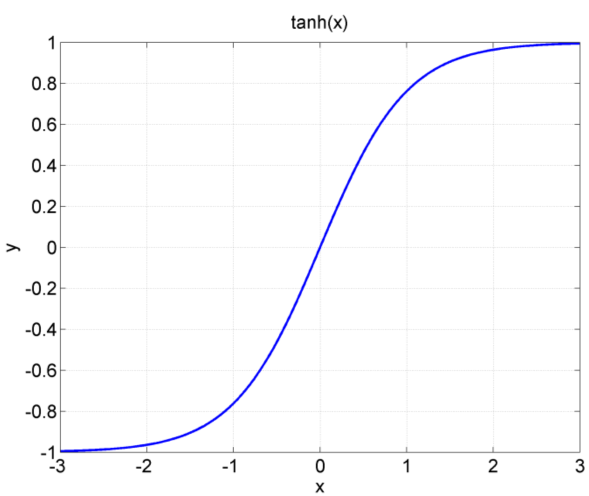
$$\omega^2 = \frac{\sigma k^3 + g(\rho_1 - \rho_2)k}{\rho_2 + \rho_1 \operatorname{cotanh}(kD)}$$

fluid densities

depth

For waves on a water, air interface $\rho_1 \approx 1000\rho_2$, so ignore ρ_2

$$\omega^2 = \left(\frac{\sigma k^3}{\rho_1} + gk\right)\tanh(kD)$$



For shallow water waves $\tanh(kD) \approx kD$

$$\omega^2 = \frac{\sigma k^4 D}{\rho_1} + gk^2 D$$

For deep water waves $\tanh(kD) \approx 1$

$$\omega^2 = \frac{\sigma k^3}{\rho_1} + gk$$

Since $k = \dfrac{2\pi}{\lambda}$ the higher powers of *k* will contribute less for longer wavelengths

Hence for shallow water gravity waves $\omega^2 = gk^2 D$

e.g. waves coming ashore just before they break, or waves in a shallow river or canal

Similarly for deep water waves $\omega^2 = gk$

Note for deep water *ripples* (or 'capillary waves') this approximation is invalid.

We must use the full dispersion relation

$$\omega = \sqrt{\frac{\sigma k^3}{\rho_1} + gk}$$

Ripple phase velocity is:

$$c_p = \frac{\omega}{k} = \sqrt{\frac{\sigma k}{\rho_1} + \frac{g}{k}} = \sqrt{\frac{2\pi\sigma}{\lambda\rho_1} + \frac{g\lambda}{2\pi}}$$

Which has a *minima* at

$$c_p = \sqrt[4]{\frac{4g\sigma}{\rho_1}}$$