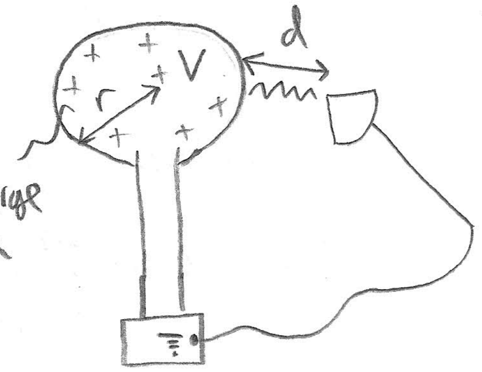
ELECTRIC FIELDS & CAPACITORS

1/ (i)



Van der Graaf generator

Spark when $\boxed{\frac{V}{d} > 3.0 \times 10^{6}\,Vm^{-1}}$

in air.

$\therefore \; V = 3.0 \times 10^{6}\,Vm^{-1} \times 0.15\,m$

$\boxed{V = 450\,kV}$

(ii) $r = 0.20\,m$. $\boxed{C = 4\pi\varepsilon_0 r}$ for a charged sphere.

$\therefore \; C = 4\pi \times 8.85 \times 10^{-12} \times 0.20 = \boxed{2.22 \times 10^{-11}\,F}$

$\boxed{Q = CV}$ $\therefore$ $Q =$ above answer $\times 450 \times 10^{3}$ C

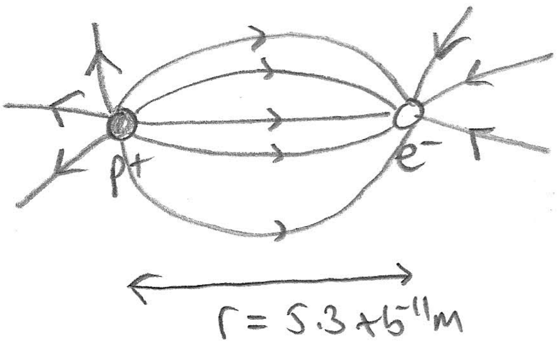
$= \boxed{1.00 \times 10^{-5}\,C}$

$E = \frac{1}{2}CV^2 \; \therefore \; E = \frac{1}{2} \times 2.2 \times 10^{-11} \times (450 \times 10^{3})^2$

$= \boxed{2.25\,J}$

(which is why a Van der Graaf is impressively high voltage but not very dangerous, since actually very little charge is stored, and not much energy is discharged in a spark).

(iii)



$\boxed{E(r) = \frac{e}{4\pi\varepsilon_0 r^2}}$ Field strength at e^- sourced by p^+.

$= \frac{1.602 \times 10^{-19}}{4\pi \times 8.85 \times 10^{-12} \times (5.3 \times 10^{-11})^2}$

$= \boxed{5.13 \times 10^{11}\,V/m}$

↳ huge!

Force on electron is $\boxed{F = eE}$

so above answer $\times 1.602 \times 10^{-19} = \boxed{8.22 \times 10^{-8}\,N}$

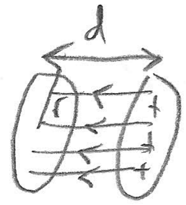
(iv) $\dfrac{F_E}{F_G} = \dfrac{\dfrac{e^2}{4\pi\varepsilon_0 r^2}}{\dfrac{G M_e^2}{r^2}} = \dfrac{e^2}{4\pi\varepsilon_0 G M_e^2}$

$$= \frac{(1.602\times10^{-19})^2}{\left[4\pi\times8.85\times10^{-12}\times6.67\times10^{-11}\times(9.109\times10^{-31})^2\right]}$$

$$= \boxed{4.17\times10^{42}}$$

So electric forces are massively stronger than gravitational forces.

For a 'reasonable' 'human sized' gravitational force, you need a planet-worth of mass.

(v) 

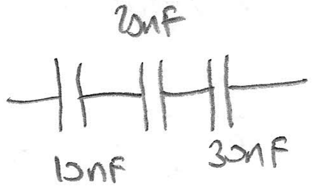
$\boxed{C = \dfrac{\varepsilon_0 \pi r^2}{d}} = 0.42\,\mu F$

$d \to d/2, \quad r \to 3r$

$\therefore C \to \dfrac{\varepsilon_0 \pi (3r)^2}{d/2} = \dfrac{18\,\varepsilon_0 \pi r^2}{d}$

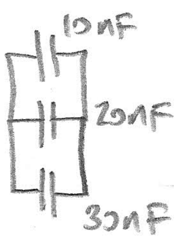
$= 18 \times 0.42\,\mu F$

$= \boxed{7.56\,\mu F}$

(vi) a) 

$C = \dfrac{1}{\frac{1}{10}+\frac{1}{20}+\frac{1}{30}}$ nF

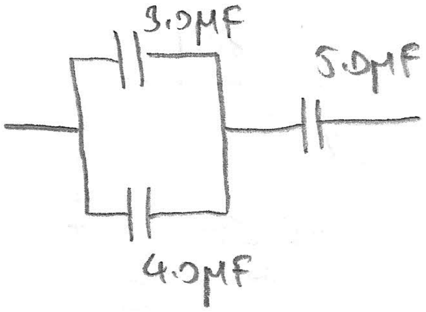
$C = \dfrac{60}{11} = \boxed{5\tfrac{5}{11}\text{ nF}}$ (5.45 nF)

b) 

$C = (10+20+30)$ nF

$\boxed{C = 60\text{ nF}}$

(vii)

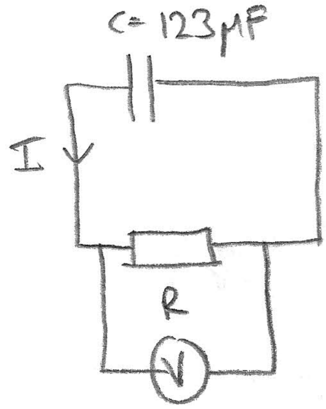


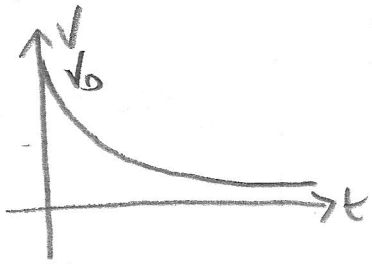
$$C = \left[\frac{1}{3+4} + \frac{1}{5}\right]^{-1} \mu F$$

$$C = \frac{35}{12} \mu F$$

$$\boxed{C = 2\tfrac{11}{12} \mu F} \quad (2.92 \mu F)$$

(viii)





$$\boxed{V = V_0 e^{-t/RC}}$$

$$\therefore \frac{V_0}{V} = e^{t/RC}$$

$$\therefore \ln\left(\frac{V_0}{V}\right) = t/RC$$

$$\therefore \boxed{t = RC \ln\left(\frac{V_0}{V}\right)}$$

$V_0 = 12.0V \quad C = 123 \times 10^{-6} F$

$t = 3.21s \quad V = 4.41V$

$$\therefore R = \frac{t}{C\ln(V_0/V)}$$

$$R = \frac{3.21}{123 \times 10^{-6} \ln\left(\frac{12.0}{4.41}\right)}$$

$$\boxed{R = 26.1 \, k\Omega}$$

Note $\boxed{RC = 3.21s}$

So after 3.21s we expect $V = \frac{V_0}{e}$

Check: $\frac{12.0}{e} = 4.41$ ✓

let $V = 1.0$:

$$\Rightarrow t = \underbrace{26070.66...}_{\text{R in calc memory}} \times 123 \times 10^{-6} \ln\left(\frac{12.0}{1.0}\right)$$

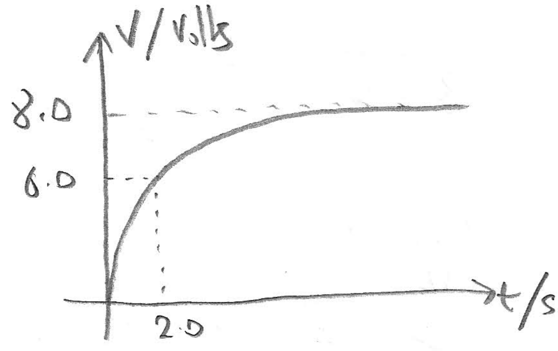
$$\boxed{t = 7.97s}$$

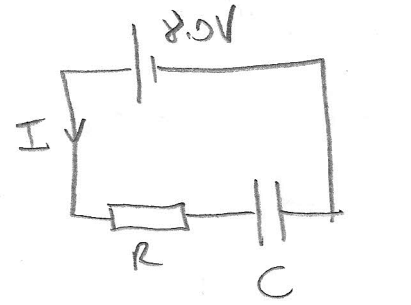
Now $\boxed{I = V/R}$ ← ohm's law

So after 2.0s: $I = \frac{12.0 e^{-2.0/3.21}}{26070.66...}$

$$= \boxed{2.47 \times 10^{-4} A}$$

(ix)





$R = 300 \times 10^{3}\ \Omega$

$$\boxed{V = V_0\left(1 - e^{-t/RC}\right)}$$

$$\therefore\ 1 - V/V_0 = e^{-t/RC}$$

$$\frac{V_0 - V}{V_0} = e^{-t/RC}$$

$$e^{t/RC} = \frac{V_0}{V_0 - V}$$

$$t/RC = \ln\left(\frac{V_0}{V_0 - V}\right)$$

$$\therefore\ \boxed{C = \frac{t}{R\ln\left(\frac{V_0}{V_0 - V}\right)}}$$

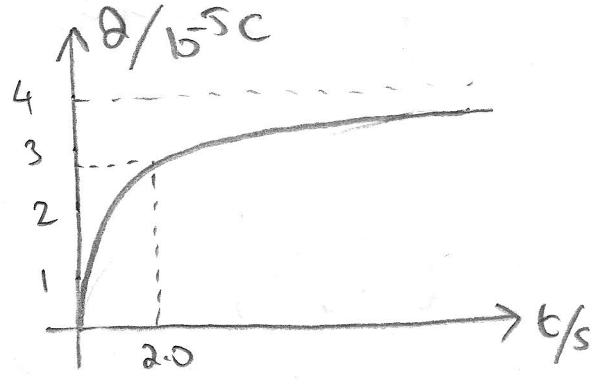
$$\text{So } C = \frac{2.0}{300\times 10^{3}\ln\left(\frac{8.0}{8.0-6.0}\right)} = \boxed{4.81\ \mu F}$$

$Q = CV$ so max charge is $4.81\times10^{-6} \times 8.0$ (C)

$= \boxed{3.85\times10^{-5}\ C}$

when $V = 6.0V$ at $t = 2.0s$

$Q = 4.81\times10^{-6}\times6.0 = \boxed{2.89\times10^{-5}C}$

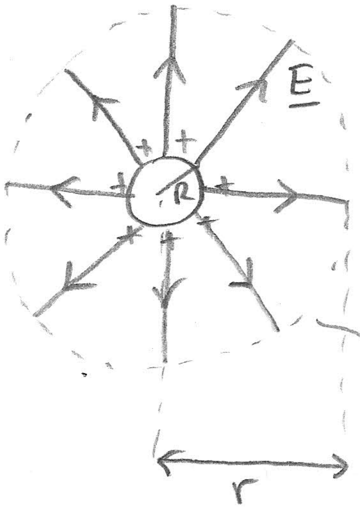


Max energy in capacitor is

$$\boxed{E = \tfrac{1}{2}CV^2}$$

$$\text{So } E_{max} = \tfrac{1}{2}\times4.81\times10^{-6}\times(8.0)^2 = \boxed{1.54\times10^{-4}\ J}$$

(x) Gauss' law : $\boxed{\int \underline{E} . d\underline{s} = \frac{Q}{\varepsilon_0}}$



Spherical "gaussian" surface S of radius r

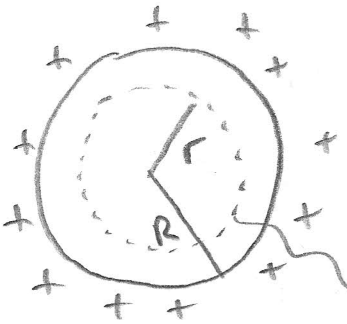
Since all field lines, by symmetry must be radial :

$$\int \underline{E} . d\underline{s} = E \times 4\pi r^2$$

$$\therefore \quad E \times 4\pi r^2 = Q / \varepsilon_0$$

$$\therefore \boxed{E(r) = \frac{Q}{4\pi \varepsilon_0 r^2}}$$

Coulomb's law.



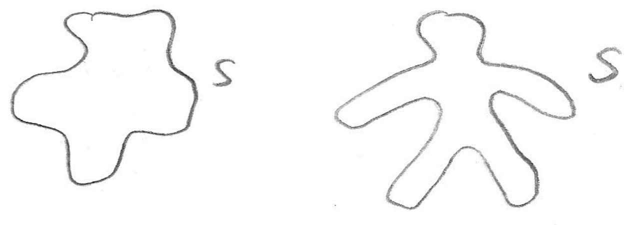
Spherical "gaussian surface S of radius r.

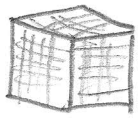
Hollow conductor

$r < R$

Gauss : $\int \underline{E} . d\underline{s} = 0$ since no charge enclosed. $\therefore$ $\boxed{E = 0 \text{ inside conductor}}$

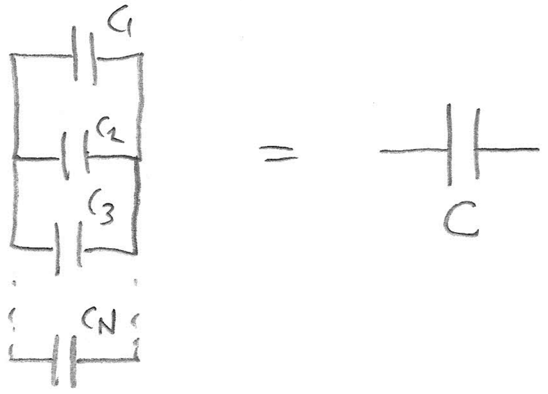
Now $\int \underline{E} . d\underline{s} = 0$ doesn't specify the surface S, only that it encloses ZERO charge.



So you can make a sealed conductive surface into a FARADAY CAGE, either a box  or a mesh suit depending on the application.

Engineers wearing a mesh suit can work safely on live high voltage equipment as the suit will be at the same potential ie no current can flow through their bodies.

(xi) Capacitors in $\boxed{\text{parallel}}$



once fully charged, each capacitor must have the same voltage between the plates. let this be V.

Now $Q = CV$

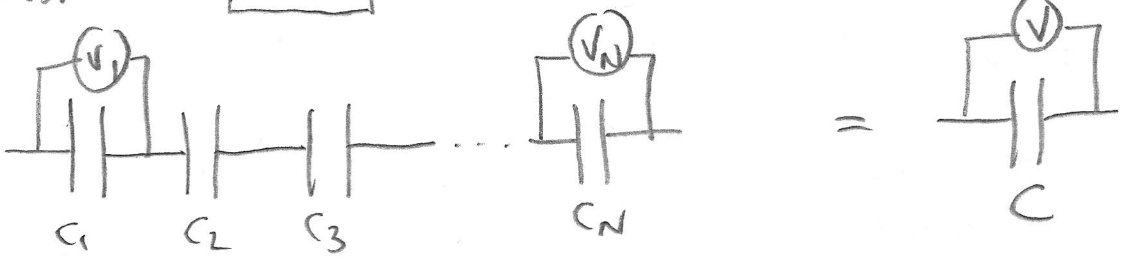
Total charge $Q = Q_1 + Q_2 + \dots Q_N$.

$Q_n = C_n V$ So $(C_1 + C_2 + \dots C_N)V = CV$

$$\therefore \boxed{C = C_1 + C_2 + \dots C_N}$$

ie capacitances <u>add</u> when wired in parallel.

Capacitors in $\boxed{\text{Series}}$



Potential differences across each capactor must sum to V.

$V_1 + V_2 + \dots V_N = V$ Now $Q_n = C_n V_n$

$$\therefore \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \dots \frac{Q_N}{C_N} = \frac{Q}{C}$$

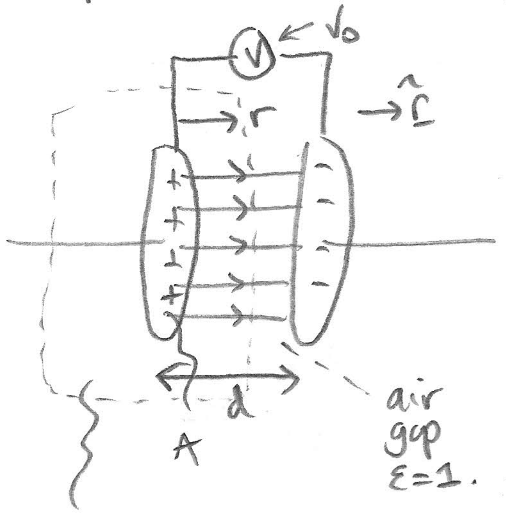
Now the charge on each capacitor plate must be the same, otherwise current would flow between the capacitors, re-apportioning charge.

$$\therefore \quad Q_1 = Q_2 = Q_3 = Q_N = Q$$

$$\therefore \quad \boxed{\frac{1}{C_1} + \frac{1}{C_2} + \cdots \frac{1}{C_N} = \frac{1}{C}}$$

ie capacitances add in reciprocals when wired in series.

(xii)

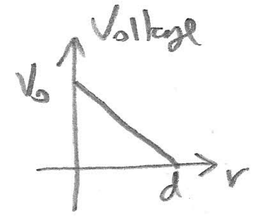


"Gaussian pill-boxes"

Electric field between plates:

$$\boxed{\underline{E} = -\frac{dV}{dr}\,\hat{\underline{r}}}$$

$$V(r) = V_0 - \frac{V_0}{d} r$$



So $\boxed{E = \frac{V_0}{d}}$

Gauss' law: $\boxed{\int_S \underline{E}.d\underline{s} = \frac{Q}{\varepsilon_0}}$

The only field is assumed to be constant, from + plate to − plate.

So $EA = Q/\varepsilon_0$

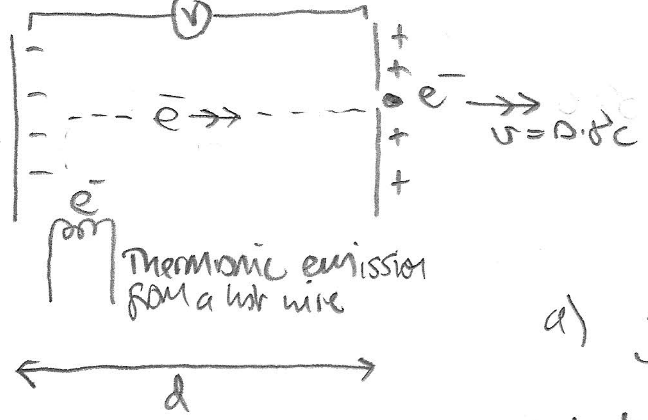
Now $\boxed{Q = CV_0}$

$$\therefore \quad \frac{V_0}{d} A = \frac{CV_0}{\varepsilon_0}$$

$$\therefore \quad \boxed{C = \varepsilon_0 A/d}$$

Capacitance of a parallel plate capacitor of area A and plate separation d.

Q2



e^- accelerated in a uniform electric field.

a) $\underbrace{eV}_{\text{Work done on } e^- \text{ by Electric field}} = \underbrace{\frac{1}{2} M_e v^2}_{\text{KE gained (classical physics)}}$

$\therefore V = \dfrac{\frac{1}{2} M_e v^2}{e}$ let $v = 0.8c$

$\therefore V = \frac{1}{2} M_e \times 0.8^2 \, c^2 / e$

$$V = \frac{\frac{1}{2} \times 9.109 \times 10^{-31} \times 0.8^2 \times (2.998 \times 10^8)^2}{1.602 \times 10^{-19}}$$

$\boxed{V = 164 \text{ kV}}$

b) Relativistic calculations:

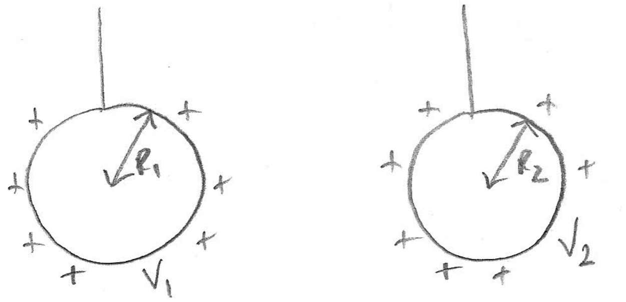
$\underbrace{(\gamma - 1) M_e c^2}_{\text{KE}} = eV$

$$\therefore V = \frac{\left[(1 - 0.8^2)^{-\frac{1}{2}} - 1\right] \times 9.109 \times 10^{-31} \times (2.998 \times 10^8)^2}{1.602 \times 10^{-19}}$$

$\boxed{V = 341 \text{ kV}}$ ie about 2.1 times the expected classical voltage.

$(\gamma = 1\frac{2}{3})$

3/



a) $Q_1 = C_1 V_1$

$$\boxed{Q_1 = 4\pi\varepsilon_0 R_1 V_1}$$

$Q_2 = C_2 V_2$

$$\boxed{Q_2 = 4\pi\varepsilon_0 R_2 V_2}$$

$E_1 = \frac{1}{2} C_1 V_1^2$

$E_1 = \frac{1}{2} 4\pi\varepsilon_0 R_1 V_1^2$

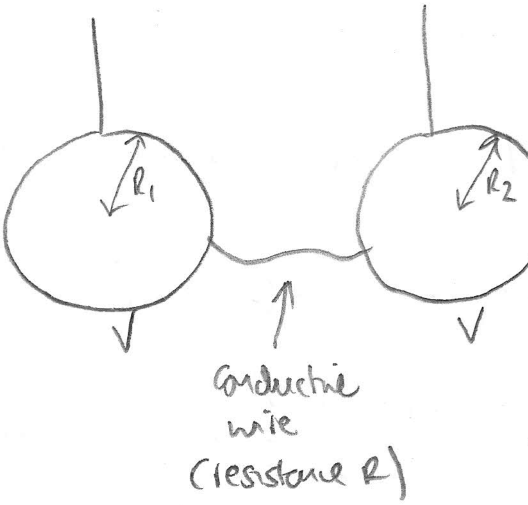
$$\boxed{E_1 = 2\pi\varepsilon_0 R_1 V_1^2}$$

$E_2 = \frac{1}{2} C_2 V_2^2$

$E_2 = \frac{1}{2} 4\pi\varepsilon_0 R_2 V_2^2$

$$\boxed{E_2 = 2\pi\varepsilon_0 R_2 V_2^2}$$

b)



Charge flows along the wire until the potential difference across the wire $= 0$.

The current in the wire

ohm's law $\rightarrow$ $I = \Delta V / R$. So $I = 0$ when $\Delta V = 0$.

Charge is conserved, so $C_1 V + C_2 V = C_1 V_1 + C_2 V_2$

$$\Rightarrow \boxed{V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}}$$

$$\therefore \boxed{V = \frac{R_1 V_1 + R_2 V_2}{R_1 + R_2}}$$

So $Q_1 = C_1 V = \boxed{\frac{4\pi\varepsilon_0 R_1}{R_1 + R_2}(R_1 V_1 + R_2 V_2)}$

$Q_2 = C_2 V = \boxed{\frac{4\pi\varepsilon_0 R_2}{R_1 + R_2}(R_1 V_1 + R_2 V_2)}$

and $E_1 = \frac{1}{2}C_1V^2 = \boxed{2\pi\varepsilon_0 R_1 \left(\frac{R_1V_1 + R_2V_2}{R_1+R_2}\right)^2}$

$$\boxed{E_2 = 2\pi\varepsilon_0 R_2 \left(\frac{R_1V_1 + R_2V_2}{R_1+R_2}\right)^2}$$

after wire connection.

c) Energy lost $\Delta E = 2\pi\varepsilon_0 \left[R_1V_1^2 + R_2V_2^2 - (R_1+R_2)\left(\frac{R_1V_1+R_2V_2}{R_1+R_2}\right)^2\right]$

$$\Rightarrow \boxed{\Delta E = 2\pi\varepsilon_0 \left[R_1V_1^2 + R_2V_2^2 - \frac{(R_1V_1+R_2V_2)^2}{R_1+R_2}\right]}$$

let $V_1 = 300\,kV$ $\quad V_2 = 400\,kV$ $\quad R_1 = 10.0\,cm$ $\quad R_2 = 7.0\,cm$

So: $\Delta E = 2\pi\varepsilon_0 \left[0.1\times(300\times10^3)^2 + 0.07\times(400\times10^3)^2 + \dots \right.$

$\left. \dots - \frac{(0.1\times300\times10^3 + 0.07\times400\times10^3)^2}{0.1+0.07}\right]$

8.85×10^{-12}

$$\boxed{\Delta E = 2.29\times10^{-2}\ J}$$

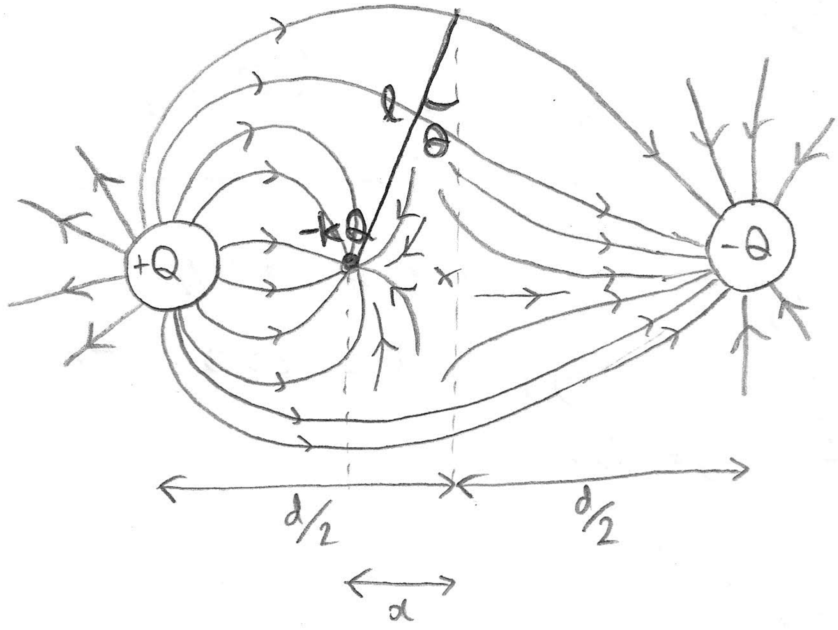
BEFORE:

$Q_1 = 3.34\,\mu C$
$Q_2 = 3.11\,\mu C$
$E_1 = 0.51\,J$
$E_2 = 0.62\,J$

AFTER:

$V = 341\,kV$
$Q_1 = 3.79\,\mu C$
$Q_2 = 2.66\,\mu C$
$E_1 = 0.65\,J$
$E_2 = 0.45\,J$
$\Delta E = 0.023\,J$

4/

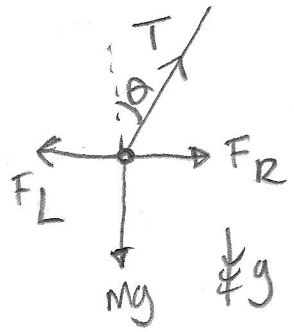


a) Electric fields must be $\perp$ to the surfaces of conductors. If there was a // component, charge would flow on the surface.

one assumes charge can flow freely in a conductor, so in equilibrium (when charge equally distributed) there can be no // field.

and no longer flowing on the conductor surface.

b)



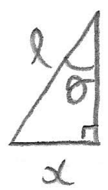
NII: vertically : $T\cos\theta - mg = 0$

Horizontally : $T\sin\theta - F_L + F_R = 0$

Coulomb : $F_L = \dfrac{kQ^2}{4\pi\varepsilon_0 \left(\frac{d}{2}-x\right)^2}$

$F_R = \dfrac{kQ^2}{4\pi\varepsilon_0 \left(\frac{d}{2}+x\right)^2}$

Also from geometry : $l\sin\theta = x$



$\therefore\ mg/\cos\theta = T$

$\therefore\ mg\tan\theta = \dfrac{kQ^2}{4\pi\varepsilon_0}\left[\dfrac{1}{\left(\frac{d}{2}-x\right)^2} - \dfrac{1}{\left(\frac{d}{2}+x\right)^2}\right]$

$$\therefore \quad \boxed{Q = \sqrt{\frac{4\pi\varepsilon_0 m g \tan\theta}{k}} \left(\frac{1}{\left(\frac{d}{2}-x\right)^2} - \frac{1}{\left(\frac{d}{2}+x\right)^2} \right)^{-\frac{1}{2}}}$$

let $m = 2.8 \times 10^{-3}$ kg , $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

$k = \frac{1}{2}$, $g = 9.81$ N/kg , $d = 20.0 \times 10^{-2}$ m

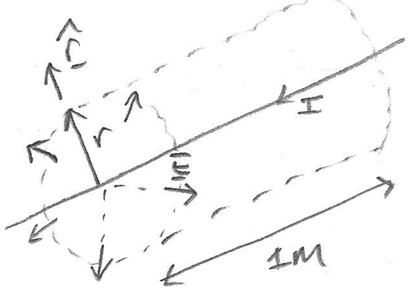
$l = 15 \times 10^{-2}$, $\theta = 30°$

$$\Rightarrow \quad \boxed{Q = 4.74 \times 10^{-8}\ \text{C}}$$

c) If this was a 10cm sphere : (10cm = R)

$$V = \frac{Q}{4\pi\varepsilon_0 R} = \frac{4.74 \times 10^{-8}}{4\pi \times 8.85 \times 10^{-12} \times 0.1} = \boxed{4260\ \text{V}}$$

5/ a) $\boxed{\text{Gauss'}:}$ consider a cylinder of unit length around current carrying wire)



$$\int_S \underline{E} \cdot d\underline{s} = \frac{Q}{\varepsilon_0} \qquad Q = \lambda$$

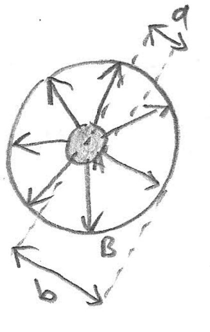
Charge / unit length

$\therefore$ Since $\underline{E} = E\hat{\underline{r}}$

$$E \times 2\pi r \times 1 = \frac{\lambda}{\varepsilon_0}$$

$$\Rightarrow \quad \boxed{E(r) = \frac{\lambda}{2\varepsilon_0 \pi r}}$$

b)



$$V_a - V_b = \int_a^b \underline{E} \cdot d\underline{r} \qquad \left(\text{since } \underline{E} = -\frac{dV}{dr}\hat{r}\right)$$

$$\therefore \quad V_a - V_b = \boxed{\frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{b}{a}\right) = V} \qquad (b > a)$$

$\therefore$ Capacitance / unit length $C = \frac{\lambda}{V}$

$\therefore$ $\boxed{C = \frac{2\pi\varepsilon_0}{\ln b/a}}$

c) Note if dielectric of relative permittivity ε is placed between conductors A, B : $\varepsilon_0 \rightarrow \varepsilon\varepsilon_0$

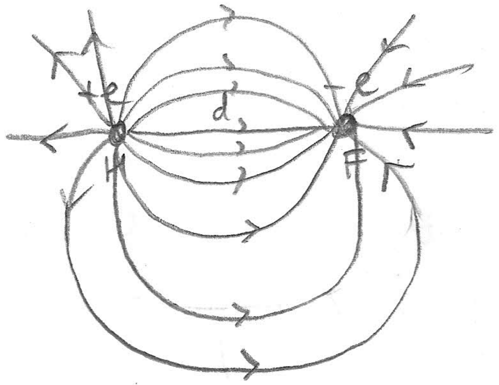
So if $a = 0.5\ \text{mm}$, $b = 3.0\ \text{mm}$, $\varepsilon = 2.25$ (polythene)

and a coaxial cable has length $l = 10.0\ \text{m}$

(capacitance) $C = \frac{2\pi \times 8.85 \times 10^{-12} \times 2.25}{\ln\left(\frac{3.0}{0.5}\right)} \times 10.0$ (F)

$\boxed{C = 6.98 \times 10^{-10}\ \text{F}}$ (0.70 nF)

6/ a)



HF (Hydrogen Fluoride) molecule.

$d = 95 \times 10^{-12}\ \text{m}$

Electric field strength at centre of molecule is:

$\hat{r} \rightarrow$

$\underline{E} = \frac{e}{4\pi\varepsilon_0 \left(\frac{d}{2}\right)^2}\hat{r} + \frac{-e}{4\pi\varepsilon_0\left(\frac{d}{2}\right)^2}(-\hat{r})$

$\therefore E = |\underline{E}| = \frac{2e \times 4}{4\pi\varepsilon_0 d^2}$

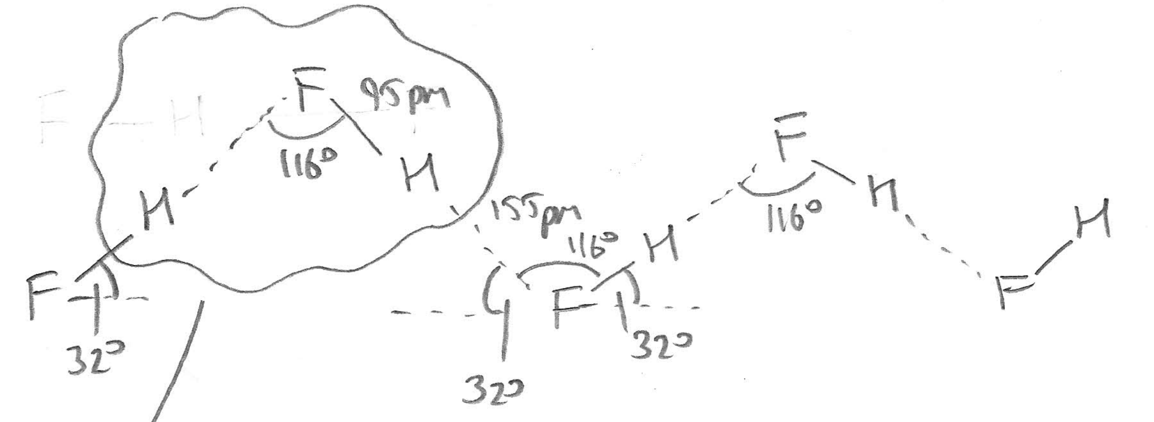
$\boxed{E = \frac{2e}{\pi\varepsilon_0 d^2}}$

So $E = \frac{2 \times 1.609 \times 10^{-19}}{\pi \times 8.85 \times 10^{-12} \times (95 \times 10^{-12})^2}$ (V/m)

$\boxed{E = 1.28 \times 10^{12}\ \text{V/m}}$

(incredibly high! Note breakdown voltage for air is 3×10^{6} V/m).

b)

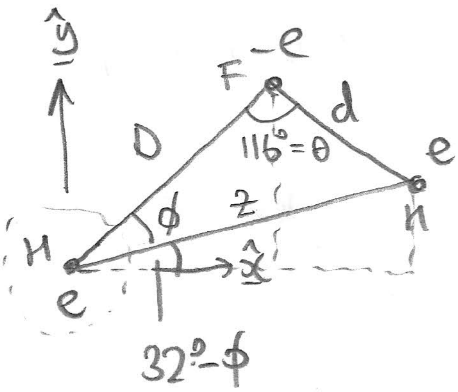


Consider just this bit for the current question

(In the liquid phase you get zig-zags of 5 → 6 molecules)

[Distance between zig-zag structures is not given

- assume to be ≫ 155 pm ?]



$d = 95$ pm $D = 155$ pm

Coordinates of F are (from H)

$$\begin{pmatrix} D\cos 32° \\ D\sin 32° \end{pmatrix}$$

Coordinates of H are (from H)

$$\begin{pmatrix} z\cos(32-\phi) \\ z\sin(32-\phi) \end{pmatrix}$$

Cosine rule: $z = \sqrt{D^2 + d^2 - 2Dd\cos 116°}$

Sine rule $\dfrac{\sin\phi}{d} = \dfrac{\sin 116°}{z}$ $\therefore$ $\phi = \sin^{-1}\left(\dfrac{d}{z}\sin 116°\right)$

$$\underline{E} \text{ @ } H = \frac{e}{4\pi\varepsilon_0 D^3}\begin{pmatrix} D\cos 32° \\ D\sin 32° \end{pmatrix} - \frac{e}{4\pi\varepsilon_0 z^3}\begin{pmatrix} z\cos(32°-\phi) \\ z\sin(32°-\phi) \end{pmatrix}$$

$$\Rightarrow \boxed{\underline{E} = 1.0 \times 10^{+10} \text{ V/m} \begin{pmatrix} 1.99 \\ 2.71 \end{pmatrix}}$$

{ I wrote a short MATLAB script to evaluate this }