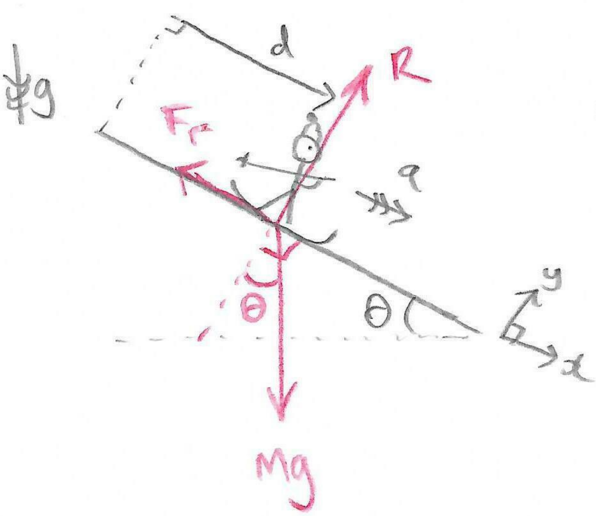
BPhO 1 2023 Dr F SOLUTIONS

a) A skier of mass m moves from rest down a slope of angle θ to the horizontal. The skier experiences a constant resistive force, F_r. At time t they have travelled a distance d down the slope. Obtain an expression for F_r in terms of the quantities given, and the acceleration due to gravity g.

[3]

DRAW A DIAGRAM FIRST ← Physics top tip!

ie "DEFINE RELATIONSHIPS BETWEEN θ, t, d, g and F_r VISUALLY"



Newton II:
(Mass × acceleration = vector sum of force)

// Slope (x direction): $\boxed{Ma = Mg\sin\theta - F_r}$ ①

⊥ Slope (y direction): $\boxed{0 = -Mg\cos\theta + R}$ ②

Since F_r is constant, ① ⇒ constant acceleration a.

∴ from kinematics, $\boxed{d = \frac{1}{2}at^2}$ since skier starts from rest.

⇒ $\boxed{a = 2d/t^2}$

∴ ①: $F_r = Mg\sin\theta - Ma$

$F_r = Mg\sin\theta - M\left(2d/t^2\right)$

⇒ $\boxed{F_r = M\left(g\sin\theta - 2d/t^2\right)}$

[Note could find coefficient of sliding friction μ using

① $F_r = \mu R$ and $R = Mg\cos\theta$] ⇒ $d(t)$ is possible to compute.

b) A car of mass 1200 kg tows a caravan of mass 600 kg. The resistive forces acting on the car and caravan are proportional to their individual weights and their sum equals 900 N. The vehicle accelerates at $2.0\,\mathrm{m\,s^{-2}}$ on a flat road.

a $R_T = R_c + R_{caravan}$.

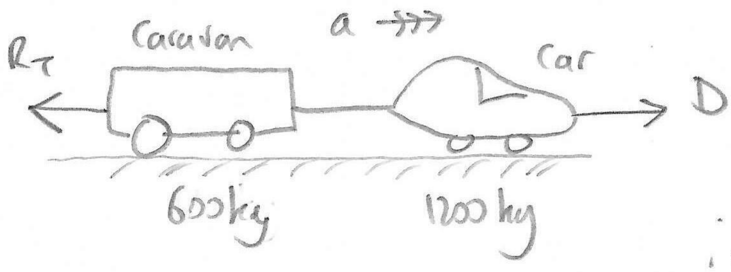
(i) What force is exerted by the engine? D

(ii) What is the tension in the tow bar? T

(iii) What is the power being converted by the engine after 5 s? P

[3]

(i) Whole system of connected objects



NII: $\boxed{(600+1200)a = D - R_T}$

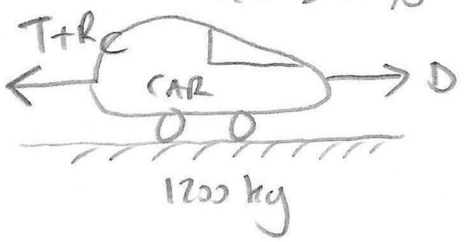
$\therefore D = 1800a + R_T$

$= 1800(2) + 900$ (N)

$= \boxed{4500\,N}$

$a = 2.0\,m/s^2$

(ii)



NII: $1200 \times 2 = D - T - R_c$

$\therefore T = 4500 - 1200 \times 2$ (N)

$- 600$

$= \boxed{1500\,N}$

$R_c = \dfrac{1200}{1200+600} \times 900\,N$

$= \boxed{600\,N}$

$\therefore R_{caravan} = 300\,N$

(iii) $\boxed{P = Dv}$

$v = 2.0 \times 5$ m/s

$= \boxed{10\,m/s}$ after 5 s.

$\therefore P = 4500 \times 10$ (W)

$= \boxed{45\,kW}$

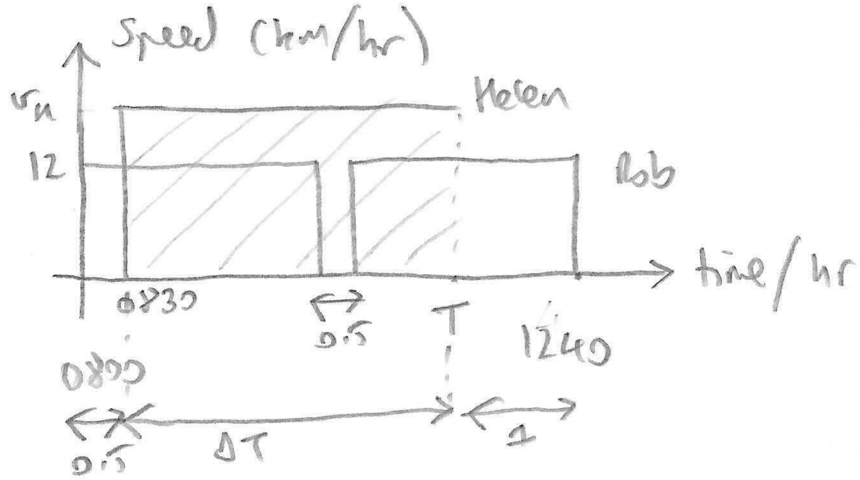
2

Area under speed vs time graph

c) The distance between village A and village B is 50 km. Helen and Robert decided to cycle from A to B.
Robert left A at 8.00 am, with a speed of $12\,\mathrm{km\,h^{-1}}$ and had a break of 30 minutes.
Helen left at 8.30 am, did not have a break and reached B an hour before Robert. What was Helen's speed?

[3]

Draw a speed vs time graph



* Then do calcs in words

* Use algebra for unknowns eg v_H.

* Robert took $\frac{50\text{ km}}{12\text{ km/h}} = \frac{25}{6}$ h of cycling

+ 0.5 hr of break. ∴ total time of $\boxed{\frac{14}{3}}$ hr

(4.7 hr) so Robert arrived at B at 1240.

* Helen travels at speed v_H without a break, arriving at time T. Note $v_H > 12$ km/h. T is $\boxed{1140}$.

so $v_H \Delta T = 50$; $\frac{14}{3} - \Delta T = 1$ $-\frac{1}{2}$ $\Rightarrow \Delta T = \frac{14}{3} - 1 - \frac{1}{2}$

$= \boxed{\frac{19}{6}}$ (hr)

Don't forget this!

$\therefore v_H = \frac{50}{19/6}$ (km/h)

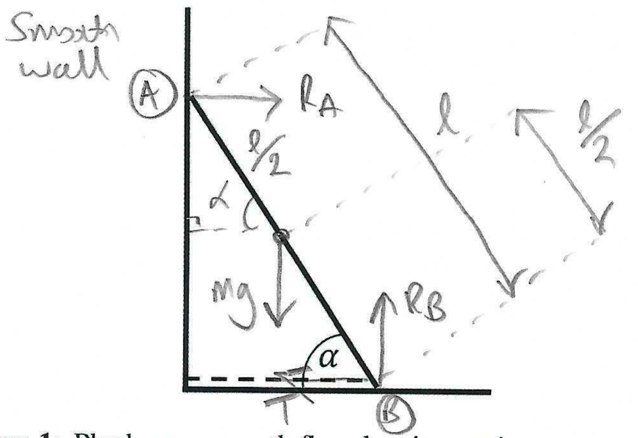
$= \frac{300}{19}$ km/h

$= \boxed{15.8\text{ km/h}} \approx 16$ km/h

3

This means $\boxed{\text{NO FRICTION}}$

d) A uniform plank of mass m stands on a smooth floor and leans against a smooth wall at an angle α to the horizontal. It is held in place by a horizontal string attached to the bottom of the ladder and to the bottom of the wall, as shown in **Fig. 1**.
What is the tension, T, in the string in terms of m, g and α?



$\not\equiv g$

Figure 1: Plank on a smooth floor leaning against a smooth wall held in place by a light string (dotted line).

[3]

EQUILIBRIUM $\Rightarrow$ * Σ of forces is zero

* Σ moments about <u>any</u> point is zero.

Want $T(m, g, \alpha)$ so take moments about (A) is a good idea. {Don't care about R_A}*

$$\circlearrowright + \quad 0 = mg \times \frac{l}{2}\cos\alpha - R_B \times l\cos\alpha + T \times l\sin\alpha$$

$$\Rightarrow \quad T = \frac{-\dfrac{mg\, l\cos\alpha}{2} + R_B\, l\cos\alpha}{l\sin\alpha}$$

$$= -\frac{mg}{2}\frac{1}{\tan\alpha} + R_B/\tan\alpha = \boxed{\frac{R_B - \frac{mg}{2}}{\tan\alpha}} \quad (1)$$

NII vertically: $\boxed{R_B = mg}$ (2) $\therefore$ $\boxed{T = \frac{mg}{2\tan\alpha}}$

or $\boxed{T = \frac{mg}{2}\cot\alpha}$ (4)

NII horizontally

* {But since $R_A = T$ we now have R_A too}.

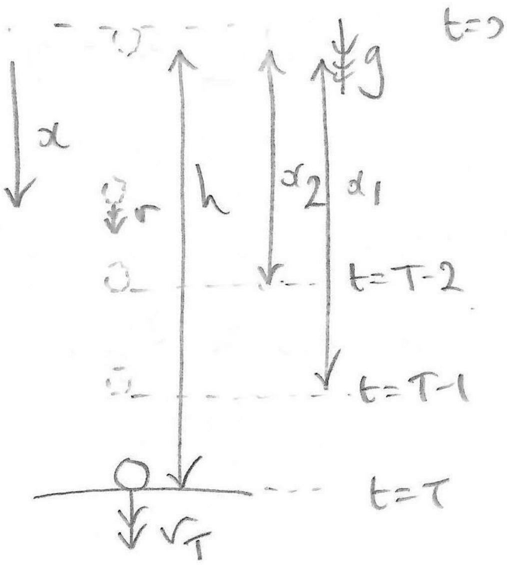
Ignore air resistance. Assume it starts from rest.

e) An object falls under gravity. The ratio of the distance fallen by the object in the last second of its fall to the distance covered in the last but one second of its fall is 3 : 2.

(i) Find the height from which the object fell, and

(ii) The speed at which it hit the ground.

[3]



Ignoring air resistance, and assuming the object falls from height h from rest

$\boxed{x = \frac{1}{2}gt^2}$ is the fall distance in t seconds

Also: $\boxed{v = gt}$

$\boxed{v^2 = 2gx}$

If T is time of fall $\Rightarrow$ $\boxed{\begin{array}{l} h = \frac{1}{2}gT^2 \\ v_T^2 = 2gh \end{array}}$

From blurb:

$$x_1 = \tfrac{1}{2}g(T-1)^2$$
$$x_2 = \tfrac{1}{2}g(T-2)^2$$

Distance fallen in last second: $h - x_1 = h - \frac{1}{2}g(T-1)^2$

Distance fallen in last but one second: $x_1 - x_2 = \frac{1}{2}g\{(T-1)^2 - (T-2)^2\}$

So if $\dfrac{h - x_1}{x_1 - x_2} = \dfrac{3}{2}$ and $h = \frac{1}{2}gT^2$

$$\Rightarrow \quad 3\{\underbrace{(T-1)^2 - (T-2)^2}_{x_1 - x_2}\} = 2\{\underbrace{T^2 - (T-1)^2}_{h - x_1}\}$$

$$3\{\cancel{T^2} - 2T + 1 - \cancel{T^2} + 4T - 4\} = 2\{\cancel{T^2} - \cancel{T^2} + 2T - 1\}$$

$$3(2T - 3) = 2(2T - 1)$$

$$2T = 7$$

$\therefore \boxed{T = 7/2 = 3.5\,(s)}$

↓ PTO

5

So if $T = 7/2$, $h = \frac{1}{2}gT^2 = \frac{1}{2}g \times \frac{49}{4}$

(3.5s)

$= \frac{49}{8}g$

$\simeq \frac{49}{8} \times 9.8$ (m)

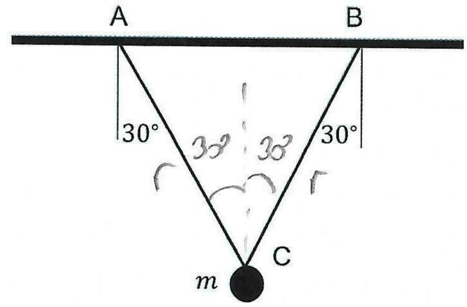
$\simeq \boxed{60\ \mathrm{m}}$ to 2.s.f.

$v_T = gT = 9.8 \times 3.5$ (m/s)

$= \boxed{34.3\ \mathrm{m/s}}$ or 34 m/s to 2 s.f.

☺ ✱ trick for solving this problem is to use a polar (r, θ) coordinate system for part (ii)

f) A mass m is suspended from a horizontal rod by two identical wires of negligible weight, each at angle $\theta = 30°$ to the vertical when attached to points A and B on the rod as shown in **Fig. 2**. The tension in each wire is T.



$r = AC = BC$

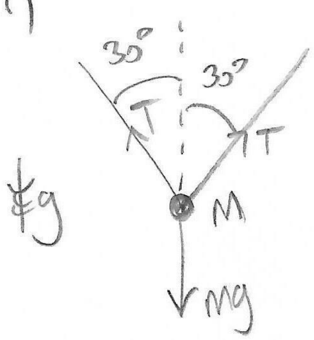
Figure 2: A mass suspended from two wires.

(i) Obtain an expression for T, the tension in each wire, in terms of m and g.

(ii) The wire BC is now cut. At the same instant the tension in wire AC, T_{AC}, will change as the system is now in motion. What is the ratio $\frac{T_{AC}}{T}$?

[3]

(i)

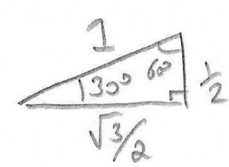


Free body (ie force) vector diagram.

if in EQ: $2T\cos 30° = mg$

$\therefore T = \frac{mg}{2\cos 30°}$

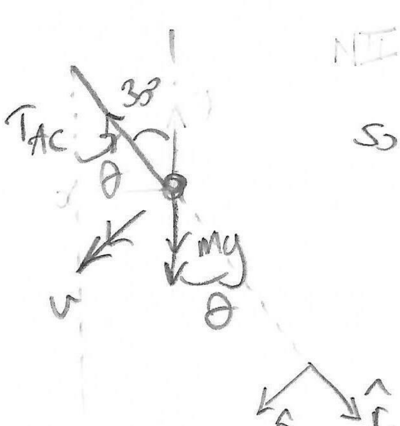
$\boxed{T = \frac{mg}{\sqrt{3}}}$



$\cos 30° = \sqrt{3}/2$

✱

(ii) At the instant that BC is cut, $T_{AC} > 0$ so motion will be a vertical circle of radius r.



So NII:

$$\boxed{\begin{array}{ll} \text{radially:} & \frac{mv^2}{r} = mg\cos\theta - T_{AC} \\ \text{tangentially} & mr\ddot{\theta} = -mg\sin\theta \end{array}}$$

Now when $\theta = \theta_0 = 30°$, $v = 0$

So at this instant $T_{AC} = mg\cos 30°$

$\Rightarrow \boxed{T_{AC} = mg\sqrt{3}/2}$

So $\frac{T_{AC}}{T} = \frac{\sqrt{3}}{2} \times \sqrt{3} = \boxed{3/2}$

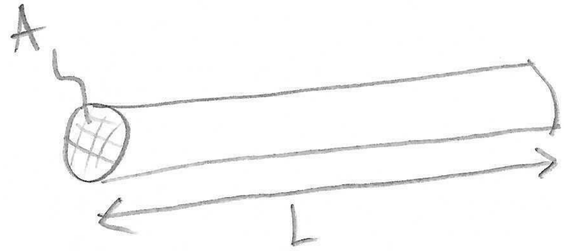
⑥

L_1 M_1 R_1 L_2

g) The resistance of a copper wire 1 m long with a mass of 1 g is 0.15 Ω. Find the length of a wire of the same material with a mass of 1000 kg and a resistance of 6000 Ω.

M_2 R_2 **[3]**

If same density, same resistivity



$$\boxed{R = \rho_E L / A}$$ resistance

ρ_E is resistivity

$$\boxed{M = \rho L A}$$ mass

ρ is density

So $R_1 = \rho_E L_1 / A_1$ $\qquad M_1 = \rho L_1 A_1$

$R_2 = \rho_E L_2 / A_2$ $\qquad M_2 = \rho L_2 A_2$

Note $A_1 \neq A_2$

So $\dfrac{R_2}{R_1} = \dfrac{L_2}{L_1} \dfrac{A_1}{A_2}$ and $\dfrac{M_2}{M_1} = \dfrac{L_2}{L_1} \dfrac{A_2}{A_1}$

$\Rightarrow \dfrac{A_1}{A_2} = \dfrac{L_1}{L_2} \dfrac{R_2}{R_1}$

$\Rightarrow \boxed{\dfrac{A_2}{A_1} = \dfrac{R_1}{R_2} \left(\dfrac{L_2}{L_1}\right)}$

$\therefore \dfrac{M_2}{M_1} = \left(\dfrac{L_2}{L_1}\right)^2 \dfrac{R_1}{R_2}$

$\Rightarrow \dfrac{L_2}{L_1} = \sqrt{\dfrac{M_2}{M_1} \dfrac{R_2}{R_1}}$

$\Rightarrow L_2 = 1.00 \sqrt{\dfrac{1000}{0.001} \times \dfrac{6000}{0.15}}$

$= \boxed{2.0 \times 10^5 \, m}$

or 200 km.

[Note this means $\dfrac{A_2}{A_1} = \dfrac{0.15}{6000} \times 2 \times 10^5 = \boxed{5}$]

But we don't know A_1.

7

✗ This is the key 'limiting case' diagram.
You don't want to find $\ell(\theta_1)$ and set $\theta_1 \to 90°$

h) A glass block, with a reflecting lower surface, is shaped as a rectangular slab but whose left and right sides are curved in the shape of quarter circles of radius R, as shown in **Fig. 3**. The base A ray of light enters horizontally from the left and passes out through the right side of the block at the same height above the base. The length of the top plane surface is ℓ.

Air $(n_1 = 1)$

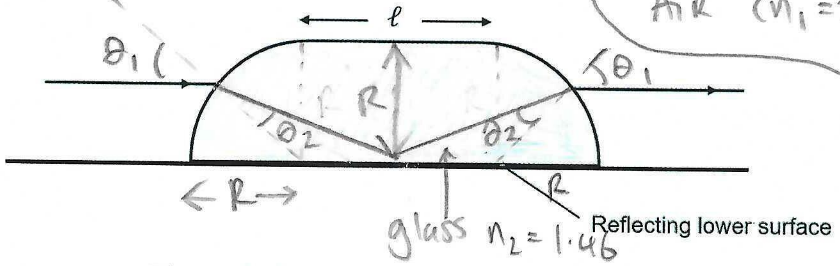


Figure 3: Light passing through a glass block.

So ray bounce point must be in the middle if symmetric

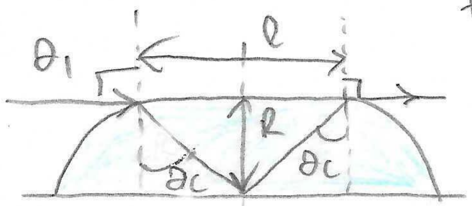
If the depth of the block is 2.0 cm (R) and the refractive index $n = 1.46$, what would be the minimum value of ℓ to satisfy this situation?

[3]

Snell: $\boxed{\sin\theta_1 \times 1.0 = \sin\theta_2 \times n}$

Critical angle θ_C is when $\theta_1 = 90°$. If $\theta_2 > \theta_C$ then total internal reflection.

✗ ie

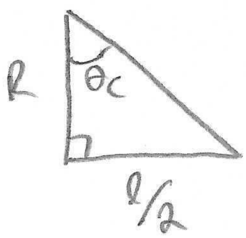


ray enters at top of block

So $n \sin\theta_C = 1 \Rightarrow \theta_C = \sin^{-1} \frac{1}{n}$

$= \sin^{-1}\left(\frac{1}{1.46}\right) = \boxed{43.2°}$

From geometry:



$\boxed{\tan\theta_C = \frac{\ell}{2R}} \Rightarrow \ell = 2R\tan\theta_C$

MIN ℓ, since larger $\ell \Rightarrow \theta_2 < \theta_C$

Now $\sin^2\theta_C + \cos^2\theta_C = 1$

$1 + \frac{1}{\tan^2\theta_C} = \frac{1}{\sin^2\theta_C}$

$\therefore \frac{1}{\tan^2\theta_C} = \frac{1}{\sin^2\theta_C} - 1$ Now $\sin\theta_C = \frac{1}{n}$ $\therefore \frac{1}{\sin^2\theta_C} = n^2$

$\therefore \tan^2\theta_C = \frac{1}{n^2 - 1} \Rightarrow \boxed{\ell = \frac{2R}{\sqrt{n^2-1}}} = \frac{2 \times 2.0\,\text{cm}}{\sqrt{1.46^2 - 1}} = \boxed{3.76}$ cm

8

P

$\alpha = 0.5$

i) An 8 W beam of light is shone on a surface at normal incidence. The surface reflects 50% of the incident light and absorbs the other 50%. $1-\alpha$

(i) What is the average force exerted on the surface by the radiation?

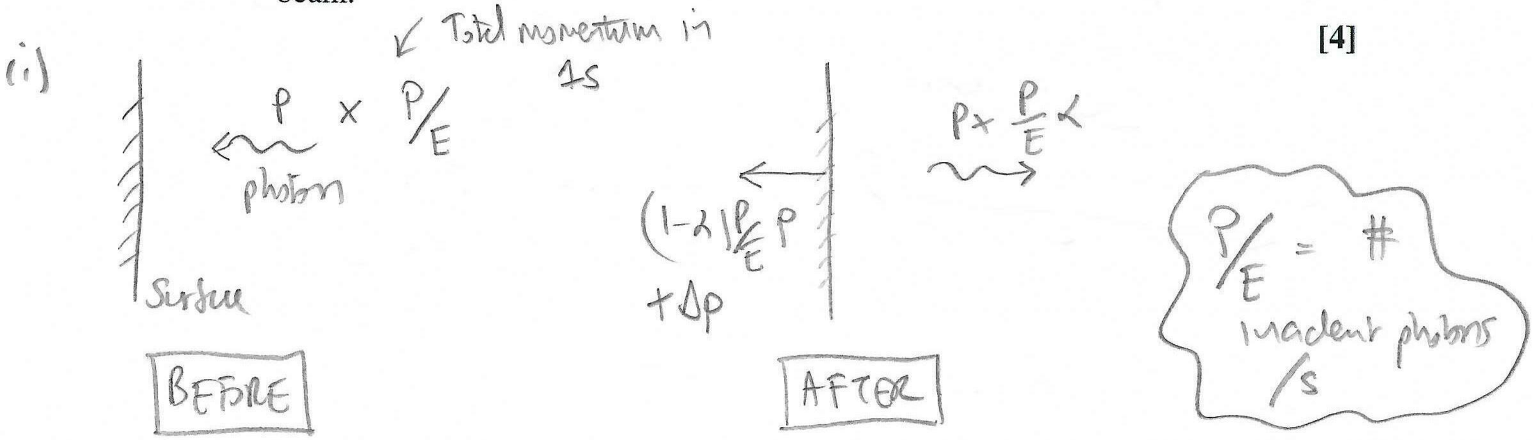
Hint: the momentum of a photon, p, is given by its energy, E, divided by the speed of light, c. i.e. $p = \frac{E}{c}$

λ A

(ii) The average wavelength of the light is 600 nm and the beam covers an area of 12 cm^2 when it is incident on the surface. Calculate the volume density of photons in the beam.

[4]

(i)



* let Δp be impulse to the surface in 1s (which means Δp is the rate of change of momentum and hence average force).

* Note absorbed photons have momentum $(1-\alpha)\frac{P}{E}p$

$\Rightarrow$ so by conservation of momentum:

$$(1-\alpha)p\frac{P}{E} + \Delta p - p\frac{P}{E}\alpha = p\frac{P}{E}$$

using $p = E/c \Rightarrow p\frac{P}{E} = P/c$

$$\therefore \Delta p = \frac{P}{c}(1+\alpha-1+\alpha) = \boxed{\frac{2\alpha P}{c}}$$

Sneaky!

BUT Can't separate surface and its absorbed photons, so total momentum per second transferred is $\underbrace{\frac{2\alpha P}{c}}_{\Delta p} + (1-\alpha)\frac{P}{c}$

$$= \boxed{(1+\alpha)P/c}$$

↓ PTO

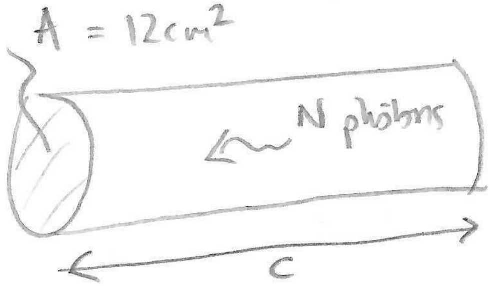
9

So total force on surface is $(1+\alpha)P/c$

$= (1+0.5) \times 8/3\times10^{8}$ (N)

$= \boxed{4\times10^{-8}\,\text{N}}$

(ii) Consider 1s of light. N photons is $P \times 1s / E$



$E = hc/\lambda$ (de Broglie)

So $\boxed{N = \dfrac{P\lambda}{hc}}$

$\therefore$ if n is photons $/m^2$

$$N = ncA$$

$$\therefore \quad n = N/cA = \boxed{\frac{P\lambda}{hc^2 A}}$$

$$= \frac{8\times600\times10^{-9}}{(3\times10^{8})^2\times6.63\times10^{-34}\times12\times(10^{-2})^2}$$

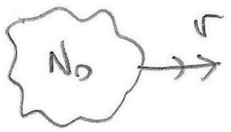
$= \boxed{6.7\times10^{13}\text{ photons}/m^3}$ photons$/m^3$

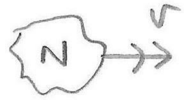
N_0

j) A short pulse of 10^8 neutrons is fired through a vacuum at a target. If the bunch of neutrons is travelling at a speed $v = 2200\ \mathrm{m\,s^{-1}}$ and the half life of a neutron is 880 s, how many neutrons will decay whilst travelling a distance of 11 m towards the target?

ΔN

[4]



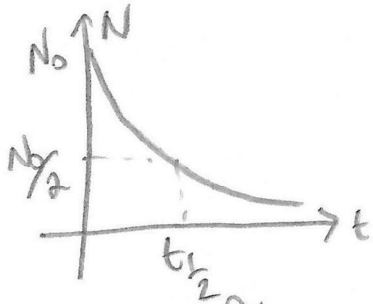


$N = N_0 - \Delta N$

$t = \frac{11}{2200}$ s is time taken to travel 11 m.

$$\boxed{N = \frac{N_0}{2^{t/t_{1/2}}}}$$

So $N = \dfrac{10^8}{2^{\frac{11}{2200 \times 880}}}$



[or $N = N_0 e^{-\lambda t}$ where $\lambda = \frac{\ln 2}{t_{1/2}}$]

$\therefore \Delta N = N_0 - N$

$= N_0\left(1 - 2^{-t/t_{1/2}}\right)$

$= 10^8\left(1 - 2^{-\frac{11}{2200 \times 880}}\right)$

$= \boxed{394 \text{ neutrons}}$

(or 390 to 2 s.f.)

[MS suggests using an approximation, since $\frac{\Delta N}{N_0} \ll 1$ but $2^{\frac{1}{176000}}$ is ok for most calculators].

↑ But easy enough if you use $N = \frac{N_0}{2^{t/t_{1/2}}}$ rather than $N_0 e^{-\lambda t}$.

(10)

k) A smoothly hinged rod of mass m rests, at angle θ to the vertical, on a smooth cylinder of radius r which sits on the floor. The hinge is set on a vertical wall at a height of $3r$ above the floor, and the lower end of the rod is at a height r above the ground when the cylinder is in the corner of the wall and the floor, as shown in **Fig. 4**.

(i) Determine the value of $\tan\theta$.

It may help to know the identity, $\tan\theta = \dfrac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$.

DRAW THE TRIANGLES

(ii) What is the length ℓ of the rod in terms of r? Give your answer as a fraction.

(iii) A light horizontal thread is attached to the rightmost point on the surface of the cylinder and it is pulled slowly to the right until the tension in the thread reduces to zero. What is the minimum amount of work that needs to be done by the thread? ☆

(iv) As the cylinder is pulled away from the wall, the angle θ increases from its minimum value θ_0, which is illustrated in **Fig. 4**, to its maximum value θ_{max}, before decreasing again. What is the ratio $\dfrac{\cos\theta_0}{\cos\theta_{max}}$?

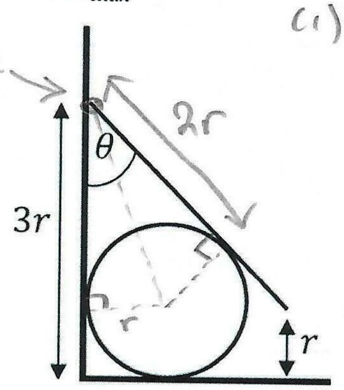
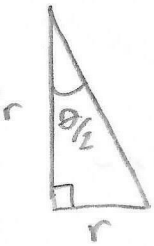


Figure 4: A hinged rod resting on a cylinder.

(i)



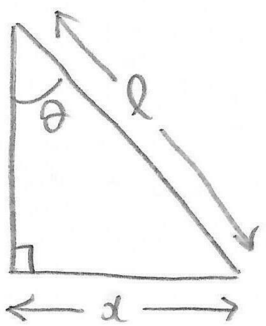
$\tan\frac{\theta}{2} = \frac{1}{2}$

$\therefore \tan\theta = \dfrac{2(\frac{1}{2})}{1-(\frac{1}{2})^2} = \dfrac{1}{3/4}$

$\Rightarrow \boxed{\tan\theta = 4/3}$

[4]

(ii)



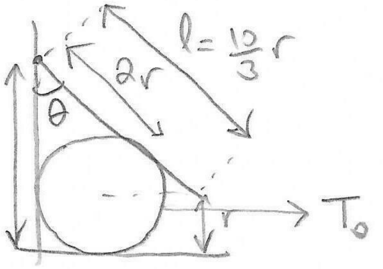
$x = 2r\tan\theta = \boxed{\frac{8}{3}r}$

Pythagoras: $\ell = \sqrt{4r^2+x^2}$

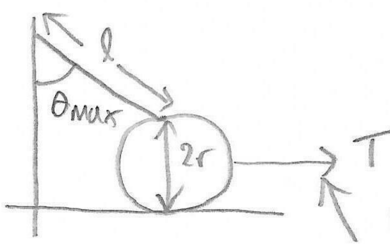
$\therefore \ell = \sqrt{4+\frac{64}{9}} \times r$

$= \sqrt{\frac{100}{9}}\, r = \boxed{\frac{10}{3}r}$

(iii)



$\longrightarrow$



(11) when the tip of the rod is directly above the cylinder, the height gain is r.

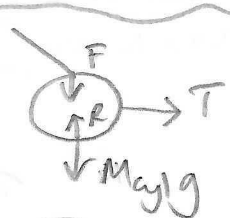
$\downarrow$

This means the centre of mass of the rod, half way between the tip and the hinge, must raise by $\frac{1}{2}r$. $\therefore$ work done is $\boxed{\frac{1}{2}Mgr}$ ie change in GPE of rod.

This is a minimum, since some KE of the cylinder will need to be achieved (one assumes this is lost to rolling resistance)

↑ If cylinder stationary when string tension $\to 0$.

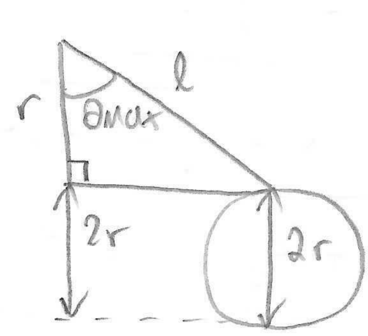
* Question is, why does



correspond to $T \to 0$.

I guess all forces on the cylinder (if we ignore friction) act along the same vertical line through the centre of the cylinder $\Rightarrow T = 0$.

(iv)

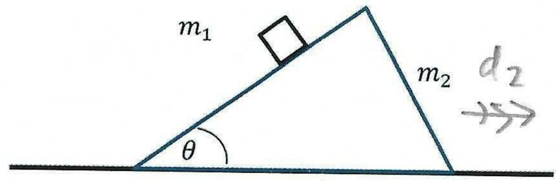


So $l\cos\theta_{max} = r$

From (ii): $l\cos\theta = 2r$

$\therefore \boxed{\dfrac{\cos\theta}{\cos\theta_{max}} = 2}$

l) An object of mass m_1 slides down the smooth sloping surface of a wedge of mass m_2, as shown in **Fig. 5**. The angle of the slope is $\theta = 30°$ to the horizontal. The wedge sits on a smooth horizontal surface.



Assume wedge accelerates at a_2 m/s^2 left to right

Figure 5: A block of mass m_1 sliding down the smooth face of a wedge of mass m_2 that sits on a smooth horizontal surface.

$\boxed{\theta = 30°}$

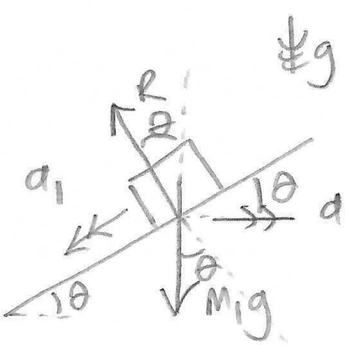
(i) Mark on the forces on two free-body diagrams.

The block's acceleration can be resolved into two components; one is down the slope, which would be the case if the wedge was fixed, and a second horizontal component so that it remains in contact with the accelerating wedge.

(ii) Resolve the forces on the sliding object normal to the slope. a_2

(iii) Hence or otherwise, obtain an expression for the acceleration of the wedge in terms of m_1, m_2 and g.

[4]

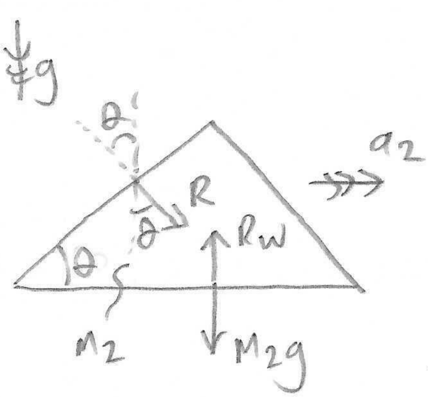


* Note the block is assumed to remain in contact with the wedge, so it accelerates a_2 left to right AND a_1 down the wedge slope *

Forces on the block

NII ⊥ slope {only a_2 contributes to this component of mass + acceleration} ↖ resolving in any other direction will involve a_1 and a_2.

$$\boxed{m_1 a_2 \sin\theta = M_1 g\cos\theta - R} \quad ①$$



NII horizontally: $\boxed{M_2 a_2 = R\sin\theta}$ ②

$$\therefore m_1 a_2 \sin\theta = M_1 g\cos\theta - \frac{M_2 a_2}{\sin\theta}$$

Forces on the wedge

↓ PTO

12

$$\therefore\ a_2\left(m_1\sin\theta + \frac{m_2}{\sin\theta}\right) = m_1 g\cos\theta$$

$$\therefore\ a_2\left(m_1\sin^2\theta + m_2\right) = m_1 g\sin\theta\cos\theta$$

$$\therefore\ a_2 = \frac{g\,m_1\sin\theta\cos\theta}{m_2 + m_1\sin^2\theta}$$

$$\therefore\ a_2 = \frac{g\sin\theta\cos\theta}{m_2/m_1 + \sin^2\theta}$$

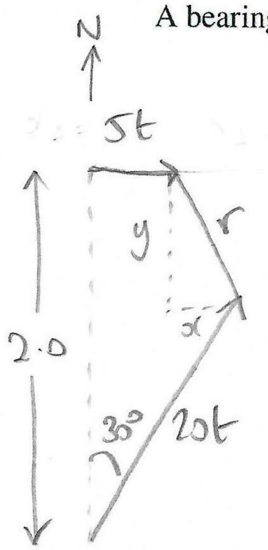
$$\therefore \text{ if } \theta = 30^\circ,\quad a_2 = \frac{g\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}{\frac{m_2}{m_1} + \left(\frac{1}{2}\right)^2}$$

$$\boxed{a_2 = \frac{g\sqrt{3}}{4m_2/m_1 + 1}}$$

* Distances in nautical miles
* t is time in hours.

m) Two ships, A and B spot each other at the moment ship B is due south of A with a range of 2.0 nautical miles. Ship A is travelling on a bearing 090° at 5.0 knots, and ship B on a bearing of 030° at 20 knots. What is their distance of closest approach? r
1 knot = 1 nautical mile per hour.
A bearing is an angle measured clockwise from 0° as due north.

[5]



Pythagoras: $r^2 = x^2 + y^2$ (1)

$20t\cos 30° + y = 2$ (2)

$20t\sin 30° = x + 5t$ (3)

$$\text{so } r^2 = \underbrace{\left(20t\left(\tfrac{1}{2}\right) - 5t\right)^2}_{x} + \underbrace{\left(2 - 20t\tfrac{\sqrt{3}}{2}\right)^2}_{y}$$

$$= (10t - 5t)^2 + (2 - 10t\sqrt{3})^2$$

$$= 25t^2 + 4 - 40t\sqrt{3} + 100 \times 3t^2$$

$$\therefore \boxed{r^2 = 325t^2 - 40t\sqrt{3} + 4}$$

Want smallest r, so $\frac{dr}{dt} = 0$ will yield this

$2r\frac{dr}{dt} = 650t - 40\sqrt{3}$, so if $r \neq 0$

$\Rightarrow \frac{dr}{dt} = 0$ when $t = \frac{40\sqrt{3}}{650} \approx 0.107$ hrs

$$\therefore r_{min}^2 = 325\left(\frac{40\sqrt{3}}{650}\right)^2 - 40\left(\frac{40\sqrt{3}}{650}\right)\sqrt{3} + 4$$

$$= \frac{48}{13} - \frac{96}{13} + \frac{52}{13}$$

$$= \frac{4}{13} \quad \therefore r_{min} = \boxed{\frac{2}{\sqrt{13}}} \approx 0.55$$

nautical miles

13

I still think the MS has errors!

i.e. radius 2.0cm

l

n) A steel tube of 4.0 cm internal diameter and 30 cm long, with a wall 0.25 cm thick, is covered externally and lined internally with copper tubes 0.20 cm thick. The three concentric tubes are firmly connected. This compound tube is placed under tension by a load and the stress produced equals $6.2 \times 10^7\,\mathrm{N\,m^{-2}}$. Determine

(i) The extension of the tube δl

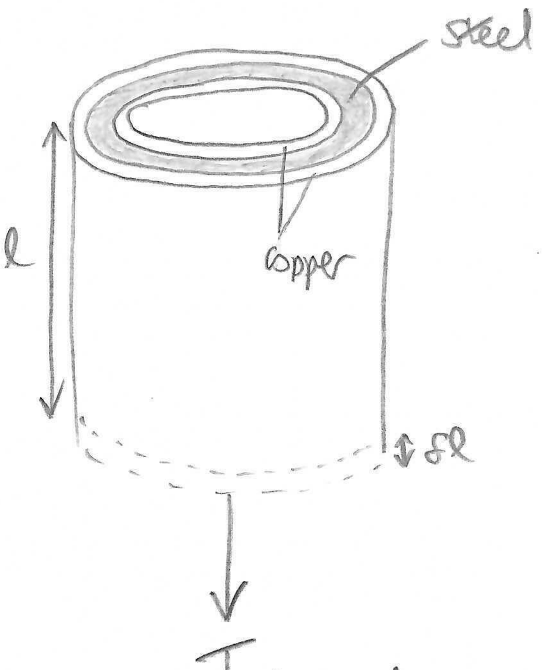
(ii) The stress in the copper tubes σ_c

(iii) The load carried by the compound tube T

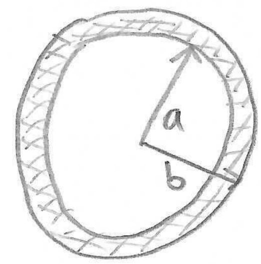
Assume cross section doesn't change under load

Young's Moduli: Steel 200 GPa
Copper 110 GPa

[5]



load force, causing extension δl



a inner radius

b outer radius

Area is $\pi(b^2-a^2)$

Copper cross section:

$$A_c = \pi(2.45^2 - 2.25^2) + \pi(2.0^2 - 1.8^2)$$

$$= \frac{17}{10}\pi \ \mathrm{cm}^2$$

$$= \boxed{1.7\pi \times 10^{-4}\ \mathrm{m}^2}$$

Not what is in the MS.

Steel cross section: $A_s = \pi(2.25^2 - 2.0^2)$

$$= \frac{17}{16}\pi \ \mathrm{cm}^2$$

$$= \boxed{1.0625\pi \times 10^{-4}\ \mathrm{m}^2}$$

Total load $T = T_c + T_s$ $\qquad \frac{T_c}{A_c} = \sigma_c \qquad \frac{T_s}{A_s} = \sigma_s$

stress.

Also: $\boxed{Y_S = \dfrac{\sigma_S}{\varepsilon}}$ $\boxed{Y_C = \dfrac{\sigma_C}{\varepsilon}}$ $\boxed{\varepsilon = \dfrac{\delta l}{l}}$ strain

Young's modulus

we are given $\sigma_{TOT} = \dfrac{T}{A_S + A_C} = 6.7 \times 10^7 \ N/m^2$

and $Y_S = 200 \ GPa$ and $Y_C = 110 \ GPa$

so $\sigma_{TOT}(A_S + A_C) = T = T_C + T_S = \sigma_C A_C + \sigma_S A_S$

$$\Rightarrow \sigma_{TOT}(A_S + A_C) = \varepsilon Y_C A_C + \varepsilon Y_S A_S$$

$$\Rightarrow \varepsilon = \frac{\sigma_{TOT}(A_S + A_C)}{Y_C A_C + Y_S A_S}$$

$$\therefore \boxed{\delta l = \frac{l \, \sigma_{TOT}(A_S + A_C)}{Y_C A_C + Y_S A_S}}$$

cancel the π factors

$$= \frac{30cm \times 6.7 \times 10^7 \left(\frac{17}{16} + 1.7\right)}{\left(110 \times 1.7 + 200 \times \frac{17}{16}\right) \times 10^9}$$

$$= \boxed{1.45 \times 10^{-3} \ m}$$ (MS is 5.16×10^{-4} m)

(ii) $\therefore \sigma_C = \varepsilon Y_C = \dfrac{1.45 \times 10^{-3}}{30 \times 10^{-2}} \times 110 \times 10^9 \ Pa$

$$= \boxed{5.3 \times 10^8 \ Pa}$$

$\therefore T = \sigma_{TOT}(A_S + A_C) = 6.7 \times 10^7 \times (1.0625 + 1.7)\pi \times 10^{-4} \ N$

$$= \boxed{5.8 \times 10^4 \ N}$$

o) A thin wire of mass $m = 6.0\,\mathrm{g}$ and length $L = 1.2\,\mathrm{m}$ is stretched to a tension T between two fixed supports. The wire is excited so that the third harmonic standing wave is formed of maximum amplitude A_{max}. In such a wire, the total KE and elastic PE are equal. The tension is $T = 45\,\mathrm{N}$ and $A_{\mathrm{max}} = 1.8\,\mathrm{cm}$.

(i) Sketch a graph of the maximum kinetic energy of the particles in the wire against their position along the length of the wire in the range 0 to L. Draw a line on your graph to show the the total energy of the particles along the wire from 0 to L.

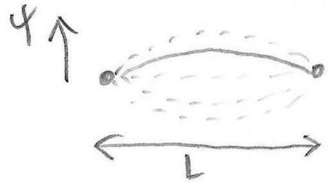
(ii) Calculate the frequency of vibration of the wire.

(iii) Calculate the total energy due to the vibration of the wire.

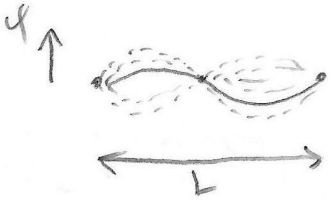
Hint: The speed of a wave v in a stretched wire is given by $v = \sqrt{\dfrac{T}{\mu}}$ where μ is the mass per unit length of the wire.

[5]

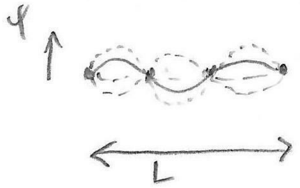
(i) Nodes at both ends of the wire, so $\boxed{L = n\lambda/2}$ $n = 1, 2, 3 \ldots$



Fundamental. 1st harmonic. $L = \lambda/2$



First overtone. 2nd harmonic. $L = 2 \times \lambda/2$



Second overtone. 3rd harmonic $L = 3 \times \lambda/2$

Wave displacement: $\boxed{\psi(x,t) = A\sin(2\pi x/\lambda)\cos\omega t}$ Since a standing wave.

Since $\lambda = \dfrac{2L}{n}$ $\Rightarrow$ $\boxed{\psi(x,t) = A\sin\left(\dfrac{\pi x n}{L}\right)\cos\omega t}$

$\Rightarrow \dfrac{2\pi x}{\lambda}$

$= \dfrac{2\pi x n}{2L}$

$= \boxed{\dfrac{\pi x n}{L}}$

velocity $\uparrow$ is $v = \partial\psi/\partial t$

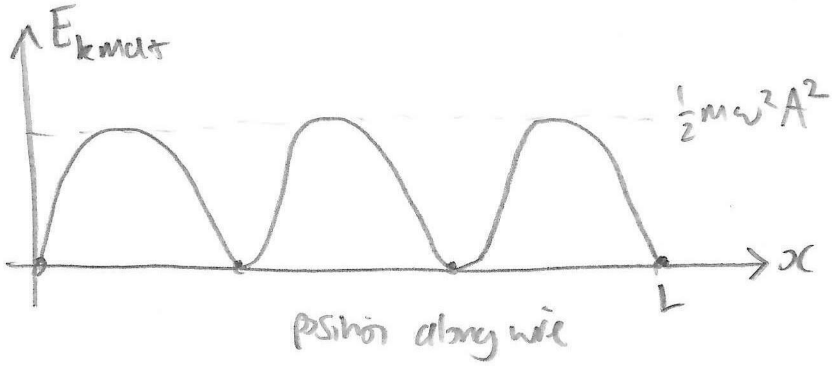
$= -\omega A \sin\left(\dfrac{\pi x n}{L}\right)\sin\omega t$

So $E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2\left(\dfrac{\pi x n}{L}\right)\sin^2\omega t$

15

So maximum KE is when $\sin\omega t = 1$ (at x)

ie $$\boxed{E_{k\max} = \frac{1}{2}m\omega^2 A^2 \sin^2\left(\frac{\pi x n}{L}\right)}$$



Take $n=3$ (3rd harmonic, 2nd overtone).

* The maximum (ie antinodes) is $\frac{1}{2}m\omega^2A^2$
* Averaging over period $2\pi/\omega$, this means average KE ~~is~~ is $\boxed{\frac{1}{4}m\omega^2A^2}$ { ie variation is symmetric between 0 and $\frac{1}{2}m\omega^2A^2$ }.

(ii) $v = \sqrt{\frac{T}{\mu}}$ $\quad v = f\lambda \quad L = \frac{3}{2}\lambda \Rightarrow \lambda = \frac{2L}{3}$

$\mu = \frac{M}{L}$ $\quad M = 6.0\times10^{-3}$ kg $\quad A = 1.8\times10^{-2}$ m

$L = 1.2$ m

$T = 45$ N

So $f = \frac{v}{\lambda} = \frac{3}{2L}\sqrt{\frac{TL}{M}} = \frac{3}{2}\sqrt{\frac{T}{ML}}$

$= \frac{3}{2}\sqrt{\frac{45}{6.0\times10^{-3}\times1.2}}$ (Hz)

$= \boxed{118.6\text{ Hz}} \approx 120$ Hz to 2sf. Now $\omega = 2\pi f$

(iii) If $E_{TOTAL} = 2\overline{E}_{KE} = \frac{1}{2}M(2\pi\times118.6)^2\times(1.8\times10^{-2})^2 = \boxed{0.54\text{ J}}$

↑ average not max

p) A cell of emf ε with an internal resistance r is connected across two resistors R_1 and R_2 in parallel, as in **Fig. 6**. Resistor R_2 is variable, whilst R_1 is fixed.

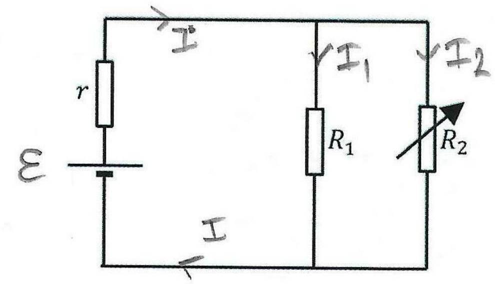


Figure 6: A circuit with two resistors connected in parallel with a cell.

Obtain expressions for

(i) the current I flowing through the cell in terms of ε, R_1, R_2, r,

(ii) the current I_2 flowing through R_2 in terms of I, R_1, R_2,

(iii) the power P_2 converted in R_2 in terms of ε, R_1, R_2, r.

(iv) By considering the term $\frac{1}{P_2}$ or otherwise, determine an expression for R_2 in terms if R_1 and r such that the power P_2 converted in R_2 is a maximum.

[5]

(i) Total resistance is $R_T = r + \dfrac{1}{\frac{1}{R_1}+\frac{1}{R_2}} = r + \dfrac{R_1R_2}{R_1+R_2}$

$$\therefore \quad I = \frac{\varepsilon}{R_T} = \frac{\varepsilon}{r+\frac{R_1R_2}{R_1+R_2}} = \frac{\varepsilon(R_1+R_2)}{r(R_1+R_2)+R_1R_2}$$

(ii) KI: $I = I_1 + I_2$

KII for right loop: $0 = I_2R_2 - I_1R_1$ ie $I_1R_1 = I_2R_2$

(ie potential across each // loop is the same).

So $I_1 = I - I_2 \therefore \quad (I-I_2)R_1 = I_2R_2$

$$\Rightarrow \quad IR_1 = I_2(R_1+R_2)$$

$$\Rightarrow \quad \boxed{I_2 = \frac{IR_1}{R_1+R_2}}$$

(iii) $P_2 = I_2^2R_2$

So $$P_2 = \frac{I^2R_1^2R_2}{(R_1+R_2)^2} = \frac{\varepsilon^2\cancel{(R_1+R_2)^2}\,R_1^2R_2}{[r(R_1+R_2)+R_1R_2]^2\,\cancel{(R_1+R_2)^2}}$$

(iv) $\left[\text{So } P_2 = \dfrac{\varepsilon^2 R_1^2 R_2}{\left[r(R_1+R_2)+R_1R_2\right]^2} \right]$

Max P_2 is the minimum of $\frac{1}{P_2}$

$$\frac{1}{P_2} = \frac{\left[r(R_1+R_2)+R_1R_2\right]^2}{\varepsilon^2 R_1^2 R_2}$$

, which is a bit easier to differentiate than P_2.

$$\frac{1}{P_2} = \frac{1}{\varepsilon^2 R_1^2}\frac{1}{R_2}\left\{ r^2(R_1+R_2)^2 + 2R_1R_2 r(R_1+R_2) + R_1^2R_2^2 \right\}$$

$$= \frac{1}{\varepsilon^2 R_1^2}\left\{ \frac{r^2(R_1^2+2R_1R_2+R_2^2)}{R_2} + 2R_1 r(R_1+R_2) + R_1^2 R_2 \right\}$$

$$= \frac{1}{\varepsilon^2 R_1^2}\left\{ \frac{r^2R_1^2}{R_2} + 2r^2R_1 + r^2R_2 + 2R_1^2 r + 2R_1R_2 r + R_1^2R_2 \right\}$$

So $$\frac{d}{dR_2}\left(\frac{1}{P_2}\right) = \frac{1}{\varepsilon^2R_1^2}\left\{ -\frac{r^2R_1^2}{R_2^2} + r^2 + 2R_1 r + R_1^2 \right\}$$

This $=0$ when $$\frac{r^2R_1^2}{R_2^2} = r^2 + 2R_1r + R_1^2 = (r+R_1)^2$$

$$\Rightarrow \quad R_2^2 = \left(\frac{rR_1}{R_1+r}\right)^2$$

$$\Rightarrow \quad \boxed{R_2 = \frac{rR_1}{R_1+r}}$$

Note this means $\boxed{\dfrac{1}{R_2} = \dfrac{1}{r} + \dfrac{1}{R_1}}$

Max power theorem

ie $R_2 = \dfrac{1}{\frac{1}{r}+\frac{1}{R_1}}$

q) Three capacitors, C_1, C_2 and C_3 are shown connected to a pair of cells of emfs ε_1 and ε_2 as shown in **Fig. 7**. Obtain an expression in terms of the capacitor values and the emfs for the potential across C_3.
It is important that you write down your initial equations clearly.

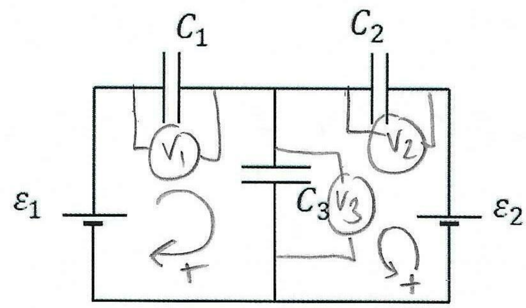
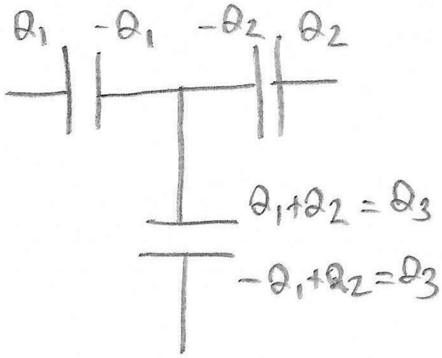


Figure 7: Three capacitors and two cells in a circuit.

[5]

Using KII round various loops:

$$\boxed{\begin{aligned} \mathcal{E}_1 &= V_1 + V_3 \quad (1) \\ \mathcal{E}_2 &= V_3 + V_2 \quad (2) \end{aligned}}$$



Since capacitors separate charge, and "in eq" no current flowing in circuit following capacitor charging

$$\Rightarrow \boxed{Q_1 + Q_2 = Q_3} \quad (3)$$

$$\boxed{\begin{aligned} Q_1 &= C_1 V_1 \\ Q_2 &= C_2 V_2 \\ Q_3 &= C_3 V_3 \end{aligned}}$$

Definitions of $C_{1,2,3}$

$$\Rightarrow \text{in } (3): \quad C_1 V_1 + C_2 V_2 = C_3 V_3 \quad (4)$$

GOAL: find $V_3(C_1, C_2, C_3, \mathcal{E}_1, \mathcal{E}_2)$

so: $C_3(1)$: $\quad C_3 \mathcal{E}_1 = C_3 V_1 + C_3 V_3 \quad (5)$

$C_3(2)$: $\quad C_3 \mathcal{E}_2 = C_3 V_3 + C_3 V_2 \quad (6)$

$(4) - (5)$: $\quad C_1 V_1 + C_2 V_2 - C_3 \mathcal{E}_1 = -C_3 V_1 \Rightarrow (C_1 + C_3) V_1 + C_2 V_2 = C_3 \mathcal{E}_1 \quad (7)$

$(4) - (6)$: $\quad C_1 V_1 + C_2 V_2 - C_3 \mathcal{E}_2 = -C_3 V_2 \Rightarrow C_1 V_1 + (C_3 + C_2) V_2 = C_3 \mathcal{E}_2 \quad (8)$

c_1 (7) $+ c_3)$: $\quad c_1(c_1+c_3)V_1 + c_1 c_2 V_2 = c_1 c_3 \varepsilon_1$

(c_1+c_3) (8) : $\quad c_1(c_1+c_3)V_1 + (c_1+c_3)(c_2+c_3)V_2 = c_3\varepsilon_2(c_1+c_3)$

$\therefore$ taking the difference to eliminate V_1 :

$$\left[(c_1+c_3)(c_2+c_3) - c_1 c_2\right] V_2 = c_3(c_1+c_3)\varepsilon_2 - c_1 c_3 \varepsilon_1$$

$$\left[\cancel{c_1 c_2} + c_3 c_2 + c_1 c_3 + c_3^2 - \cancel{c_1 c_2}\right] V_2 = c_3\left[(c_1+c_3)\varepsilon_2 - c_1\varepsilon_1\right]$$

$$(c_2 + c_1 + c_3) V_2 = (c_1+c_3)\varepsilon_2 - c_1\varepsilon_1$$

$$\therefore \quad \boxed{V_2 = \frac{(c_1+c_3)\varepsilon_2 - c_1\varepsilon_1}{c_1+c_2+c_3}}$$

See diagram of the circuit

By symmetry, we could exchange suffix $1 \to 2$

$$\Rightarrow \quad \boxed{V_1 = \frac{(c_2+c_3)\varepsilon_1 - c_2\varepsilon_2}{c_1+c_2+c_3}}$$

$\therefore$ From (1) : $\quad V_3 = \varepsilon_1 - V_1$

$$= \frac{\varepsilon_1(c_1+c_2+c_3) - (c_2+c_3)\varepsilon_1 + c_2\varepsilon_2}{c_1+c_2+c_3}$$

$$\therefore \quad \boxed{V_3 = \frac{\varepsilon_1 c_1 + \varepsilon_2 c_2}{c_1+c_2+c_3}}$$

ie a form of 'centre of mass' - like weighted average.

☺

Assume in eq.

r) A uniform rod, of length $2a$, floats partly immersed in a liquid, being supported by a string fastened to one of its ends, the other end of the string being attached to a fixed point A, as in **Fig. 8**. The density of the liquid is a factor 4/3 times that of the rod. Determine

(i) the fraction of the rod's length that will be submerged, and

(ii) the tension, T in the string.

$\downarrow g$

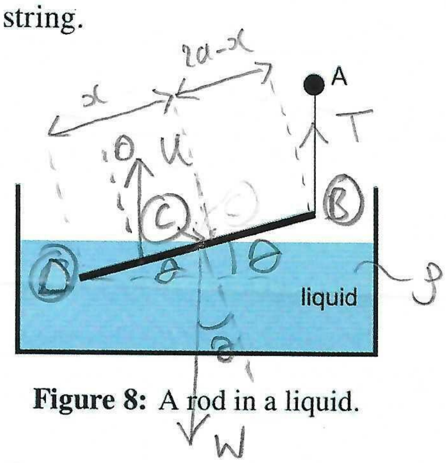
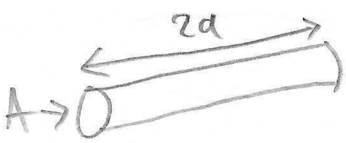


Figure 8: A rod in a liquid.

U upthrust
W weight
$\frac{4\rho}{3}$ liquid density
ρ rod density
g strength of gravity
A rod cross sectional area
C centre of rod.
[Note $DC \neq x$ in general]

[6]



Rod mass $\boxed{M = 2aA\rho}$

$\therefore \boxed{W} = Mg = \boxed{2aA\rho g}$

use $\alpha = \frac{4}{3}$

$U =$ weight of fluid displaced (Archimedes' principle) $= \boxed{Ax\alpha\rho g}$

(i) want to find $\frac{x}{2a}$, but not T yet $\rightarrow$ so take moments about B i.e. end of rod where string is attached.

$\therefore 0 = -W\cos\theta \times (2a - x) + U\cos\theta \times \left(2a - \frac{x}{2}\right)$

Since expect upthrust to act half way along submerged rod section (of length x).

$$\Rightarrow \underbrace{2a\cancel{A\rho g}}_{W}(2a-x) = \underbrace{\cancel{A}x\alpha\cancel{\rho g}}_{U}(2a - x/2)$$

$$\Rightarrow 4a^2 - 2ax = 2a\alpha x - \alpha x^2/2$$

$$\Rightarrow \frac{\alpha x^2}{2} - 2ax(1+\alpha) + 4a^2 = 0$$

$\downarrow$

$$\therefore\ x^2 - \frac{4ax}{\alpha}(1+\alpha) + \frac{8a^2}{\alpha} = 0$$

$$\left(x - \frac{2a(1+\alpha)}{\alpha}\right)^2 - \frac{4a^2(1+\alpha)^2}{\alpha^2} + \frac{8a^2}{\alpha} = 0$$

$$\text{So}\quad x = \pm\sqrt{\frac{4a^2(1+\alpha)^2}{\alpha^2} - \frac{8a^2}{\alpha}} + \frac{2a(1+\alpha)}{\alpha}$$

$$\therefore\ \boxed{x = \frac{2a(1+\alpha)}{\alpha}\left(1 \pm \sqrt{1 - \frac{2\alpha}{(1+\alpha)^2}}\right)}$$

Must be −ve if $\alpha > 1$ and $x < 2a$

$$\left[\frac{8a^2}{\alpha} \times \frac{\alpha^2}{4a^2(1+\alpha)^2} = \frac{2\alpha}{(1+\alpha)^2}\right]$$

If $\alpha = 4/3$: $\dfrac{1+4/3}{4/3} = 7/4$

$$1 - \frac{2\times 4/3}{(1+4/3)^2} = \frac{25}{49}$$

$$\therefore\ x = 2a \times \frac{7}{4}\left(1 \pm \sqrt{25/49}\right)$$

$$x = \frac{7a}{2}\left(1 \pm 5/7\right) = \underset{(-)}{a},\ \underset{(+)}{6a}$$

clearly $x < 2a$ so $\boxed{x/2a = \frac{1}{2}}$

So if $\rho = 4/3$, the weight at C really does act at the point where the rod is submerged.

↓

To find T, use $T + U = W$ (NII) or take moments about centre of rod

$T = W - U$

$= 2aA\rho g - \alpha A x \rho g$

$= 2aA\rho g \left(1 - \frac{\alpha x}{2a}\right)$

So in general: $\boxed{T = 2aA\rho g \left(1 - (1+\alpha)\left(1 \pm \sqrt{1 - \frac{2\alpha}{(1+\alpha)^2}}\right)\right)}$

But if $x = a$, $\alpha = 4/3$

$T = 2aA\rho g\left(1 - \frac{4}{3}\frac{1}{2}\right) = \boxed{\frac{2}{3}a\rho A g}$

So $T/W = \dfrac{\frac{2}{3}a\cancel{\rho A g}}{2a\cancel{A\rho g}} = \frac{1}{3}$.

s) Two balls A and B of masses and volumes m_A, V_A and m_B, V_A respectively are attached to the ends of a light thread. The thread is hung over a freely moving pulley above a beaker of water. The pulley is lowered so that B is fully submerged and ball A floats on the water. $V_A = 1000\,\text{cm}^3$, $m_A = 500\text{g}$, $V_B = 50\,\text{cm}^3$ and $m_B = 390\,\text{g}$.

(i) Sketch a diagram showing the forces on each object in the system.

(ii) What fraction f of the volume of A is submerged below the water surface?

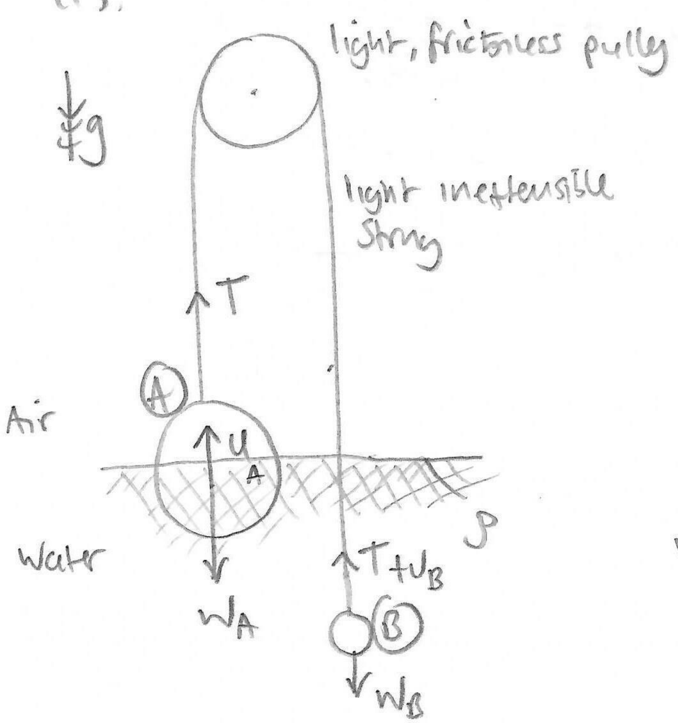
Suppose that a short section of the thread is replaced by a spring of negligible mass of spring constant $k = 100\,\text{N}\,\text{m}^{-1}$.

(iii) By how much is the spring extended?

Density of water $\rho = 1.0\,\text{g}\,\text{cm}^{-3}$

[6]

(i)



$\boxed{\text{FORCES}}$

T tension in String

U_A upthrust on Mass A

$\boxed{U_A = \rho g f V_A}$

U_B upthrust on mass B

$\boxed{U_B = \rho g V_B}$

Weights: $\boxed{W_A = M_A g}$ $\boxed{W_B = M_B g}$

(ii)

Assume System is in equilibrium, $\therefore$ by NII (ie vector sum of forces = 0)

$(A): \quad 0 = T + \rho g f V_A - M_A g \quad (1)$

$(B): \quad 0 = T + \rho g V_B - M_B g \quad (2)$

$(1) - (2): \quad 0 = \rho g (f V_A - V_B) + (M_B - M_A) g$

$\Rightarrow \quad f V_A - V_B = \dfrac{M_A - M_B}{\rho}$

$\therefore \boxed{f = \dfrac{V_B}{V_A} + \dfrac{M_A - M_B}{\rho V_A}}$

$\therefore f = \dfrac{50}{1000} + \dfrac{500 - 390}{1.0 \times 1000}$

$= \boxed{0.16}$

$(4/25)$

(iii) If the string has a short section of Hooke's spring, if extension is x, $T = kx$.

$\rho V_A = 1\,\text{kg}$

So $x = T/k$

Now from (1): $T = M_A g - \rho g f V_A$

$= 500\times 10^{-3} \times 9.8 - 9.8 \times 0.16 \times 1 \;(\text{N})$

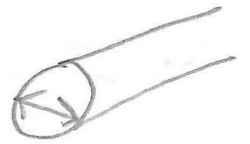
$= (0.5 - 0.16) \times 9.8 \;\text{N}$

$= 3.33\,\text{N}$.

So if $k = 100\,\text{N/M}$ $\Rightarrow$ $x = \dfrac{3.33\,\text{N}}{100\,\text{N/M}}$

$= 0.033\,\text{M}$

or $\boxed{3.33\,\text{cm}}$



2a 2a

t) A vertical wooden pole with a circular cross section of diameter 30 cm has its footing embedded in a concrete block. A loose steel guy wire, A, is fixed to the ground a distance b = 2.4 m from the base of the pole, and is attached to the pole at a height h = 12 m above the ground. It is subject to a pull to the right by tension P in a cable, attached at the same height, as shown in **Fig. 9**. This causes the wooden pole to bend a small amount to the right in the arc of a circle of radius r with $r \gg h$. The loose steel guy wire, A, then becomes tight and prevents the pole from bending any further. The bent pole stretches on one side and compresses on the other, with a neutral line down the centre of the pole. The pole has a maximum stress of 4000 N m^{-2} at one side when bent.

(i) Calculate radius r and the angle of the circular arc formed by the bent pole when the neutral line remains unstressed.

(ii) Draw a diagram to show how this angle can be used to calculate the length of the guy wire, A, when it is tight.

(iii) Calculate an approximate value for the length of slack in the guy wire when the pole is unloaded and straight and vertical.

You may assume that the bend is small enough so that the height of the pole when bent is not changed significantly.
Young's modulus for wood is 14 GPa.
Y

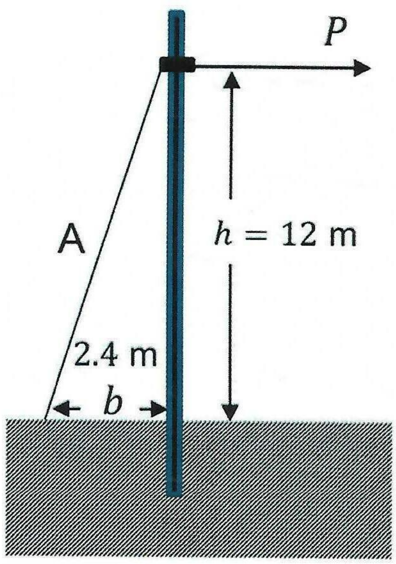
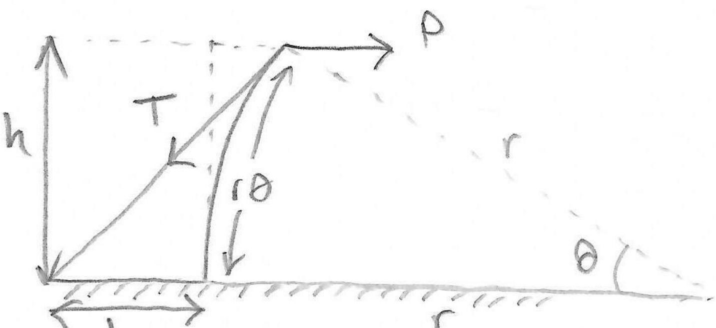


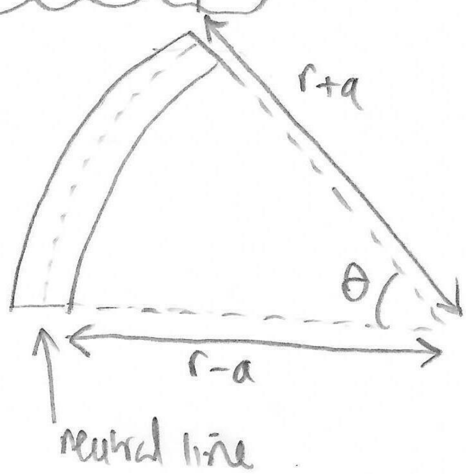
Figure 9: A pole with a guy wire A attached that will bend to the right by the tension in the cable P.

After tension P is applied:

Zoom into pole

[6]





(20)

* If outside pole arc length = $(r+a)\theta$ and inside arc length is $(r-a)\theta$ then extension will be $\boxed{2a\theta}$

* This means a strain of $\boxed{\varepsilon = \frac{a\theta}{r\theta}}$

Since we should use the $\pm$ extension from the neutral line (which is the arc of length $r\theta$).

$Y = \frac{\sigma}{\varepsilon}$, so if $\sigma = 4000\ N/m^2$ { "max stress at one side" ??

Perhaps inference is for $\sigma > 4000\ N/m^2$, neutral line is not unstressed?

$$\Rightarrow \quad Y = \frac{\sigma r}{a}$$

$$\Rightarrow \quad r = \frac{aY}{\sigma}$$

$$\Rightarrow \quad r = \frac{15\times10^{-2} \times 14\times10^{9}}{4000} \quad (m)$$

$$= \boxed{5.25 \times 10^{5} m} \qquad (525\ \underline{km}\ !!)$$

Now assume $r\theta \simeq h$ ie orignl pole length

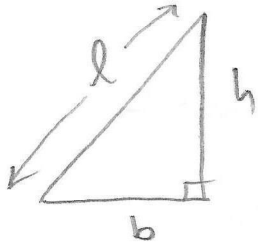
$$\Rightarrow \quad \theta \simeq \frac{h}{r} = \frac{12}{5.25\times10^{5}} \quad rad$$

$$= \boxed{2.29 \times 10^{-5}\ rad}$$

$$= \boxed{4.71}\ \text{arc seconds}$$

(3600 arc seconds in 1°).

unbacked guy wire:



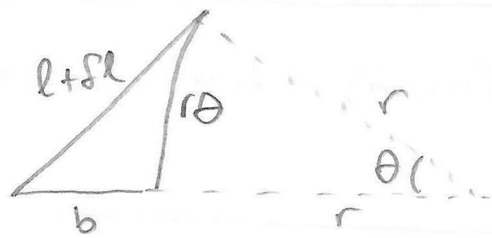
$l = \sqrt{h^2+b^2}$

Assume wire actually straight!

It probably isn't, so is this a fair calculation?

$l = \sqrt{12^2 + 2.4^2} = \boxed{12.24\ m}$

backed guy wire:
("slack taken in" increases length by δl)



cosine rule: $(l+\delta l)^2 = (b+r)^2 + r^2 - 2r(b+r)\cos\theta$

$$\Rightarrow \quad \delta l = \sqrt{(b+r)^2+r^2-2r(b+r)\cos\theta} \quad - l$$

$$\Rightarrow \delta l = \sqrt{b^2+2br+2r^2-2rb\cos\theta-2r^2\cos\theta} \quad - l$$

[b and r and θ into calc memory]

$$= \sqrt{2r^2(1-\cos\theta)+2rb(1-\cos\theta)+b^2} \quad - l$$

$$= \sqrt{2(5.25\times10^5)^2(1-\cos(2.29\times10^{-5}))+2(5.25\times10^5)(2.4)(1-\cos(2.29\times10^{-5})) + 2.4^2}$$

$- 12.24\ldots$ (m)

$r = 5.25\times10^5$
$b = 2.4$
$c = 1-\cos\theta = 2.61221\times10^{-10}$

$= \sim 7.9\times 10^{-5}$ m ✓ ☹

* Problem with Δ of small numbers. Approximation?
If $\cos\theta$ is very close to 1

$\theta = \frac{h}{r}$ and $\theta \ll 1$

So $\cos\theta \simeq 1 - \theta^2/2$

$\therefore 1 - \cos\theta \simeq \theta^2/2 = \frac{1}{2}\frac{h^2}{r^2}$

$$\delta l = \sqrt{2r^2(1-\cos\theta) + 2rb(1-\cos\theta) + b^2} - \sqrt{h^2+b^2}$$

$$\simeq \sqrt{2\cancel{r^2}\frac{1}{2}\frac{h^2}{\cancel{r^2}} + \frac{\cancel{2}rb}{\cancel{2}}\frac{h^2}{r^2} + b^2} - \sqrt{h^2+b^2}$$

$$\simeq \sqrt{h^2 + \frac{h^2 b}{r} + b^2} - \sqrt{h^2+b^2}$$

$$\simeq \sqrt{h^2(1+b/r) + b^2} - \sqrt{h^2+b^2}$$

Since $r \gg b$

$$\simeq h\sqrt{1 + b/r + b^2/h^2} - h\sqrt{1 + b^2/h^2}$$

this is clearly a small quantity!

$$= 12\left[\sqrt{1 + \frac{2.4}{5.25\times10^5} + \frac{2.4^2}{12^2}} - \sqrt{1 + \frac{2.4^2}{12^2}}\right]$$

$$= 12 \times 2.241 \times 10^{-6}$$

$$= \boxed{2.7 \times 10^{-5}\ \text{m}}$$

Don't want to expand binomially since $\frac{b}{r} \ll b^2/h^2$