

- a) An Einstein is a unit corresponding to a mole of photons. How long does it take for a 630 nm and 0.50 mW laser to release an Einstein?

[3]

- b) A loop of rope is wrapped around a tree trunk of negligible friction and clamped at A and pulled with a force F at D, as shown in **Fig. 1**.
- (i) For what angle θ is the tension in the rope at AB, AC equal to the tension in the rope AD.
 - (ii) For this condition, what is the length ℓ of the rope ABCA in terms of the radius of the tree r ? Express your answer in terms of fractions and surds (roots).

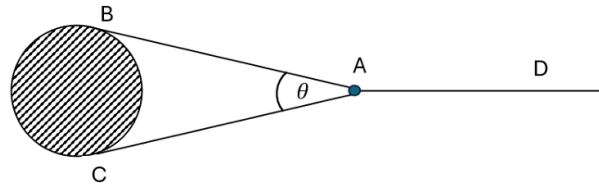


Figure 1: A tree trunk with a rope wrapped around and pulled by a force F .

[4]

- c) We can develop a simple model for traffic flow as follows; each car has a length L and travels at speed v , with the distance between the fronts of two consecutive cars being d , as shown in **Fig. 2**.

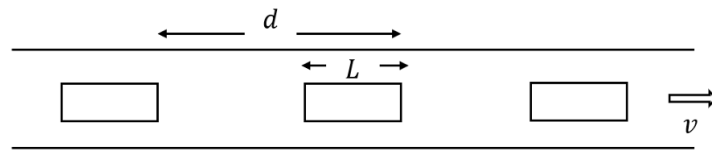


Figure 2: Flow of cars each of length L separated by distance d between car fronts.

- (i) Give the flow rate n of cars passing a point per unit time in terms of v and d .
Cars brake with a deceleration a and the drivers choose the value of d such that if the car in front were to stop instantaneously, the car behind could brake before hitting the back of the car in front.
- (ii) Obtain d in terms of v , a and L . Ignore the “thinking time”.
- (iii) Find an expression for the optimum speed v for maximum flow rate in terms of L and a .

[4]

- d) A metal sphere of radius 2.0 cm was embedded in an ice cube with a side length 10 cm floating in a bowl of water. The bowl of water was completely full (i.e., adding any more water would cause it to spill). After the ice cube had completely melted, the water level of the bowl dropped. To make the bowl full again, 60 mL of water had to be added. The densities of water and ice are 1.0 g cm^{-3} and 0.90 g cm^{-3} respectively.
- (i) Find the density of the metal.
 - (ii) Suppose the metal sphere remained embedded in the ice cube throughout the melting process. What fraction of the ice cube's volume had melted when the ice cube just sank to the bottom of the bowl?

[6]

e) A circuit consists of an ideal battery (negligible internal resistance) and three resistors. Two of the resistors labelled R_A have the same resistance and are connected in series with the battery, whilst the third, R_B is connected in parallel with the battery. If R_B and one of the R_A are swapped,

(i) Find the upper limit of the factor by which the power dissipated by the circuit can increase due to this swap.

(ii) If swapping resistors R_A and R_B halves the total dissipated power, what is the ratio

$$\frac{R_A}{R_B}$$

[6]

- f) A household filament bulb has a rating of 240 V, 60 W, whereas a car filament headlamp bulb is rated at 12 V, 60 W. The filaments of the bulbs are both tungsten, of a similar length, and operate at the same temperature in order to produce the same spectrum. Determine the ratio of the masses of the two filaments.

[3]

- g) There is a thin rectangle sheet $ABCD$ of uniform resistivity, where $AB = 2BC$, as shown in **Fig. 3**. Two measurements of its resistance using a Ohmmeter have been made. The first is made across the mid-point of AB and the edge CD (a metal band is connected along CD), giving value R_1 . The second is made between A and B , giving value R_2 . Find the ratio $\frac{R_1}{R_2}$. Show your ideas through diagrams.

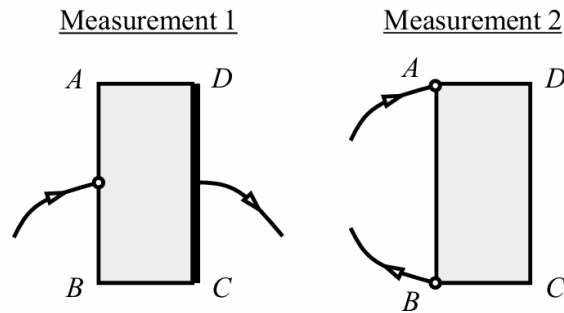


Figure 3: Resistance of a rectangular sheet

- h) An infinite tree of identical resistors R is constructed as in **Fig. 4**. Three resistors are each connected to pairs of identical resistors which themselves are each connected to pairs stretching out to infinity. If we have a current entering at the centre of the tree of resistors and the entire outer edge is grounded, what is the resistance of the tree?

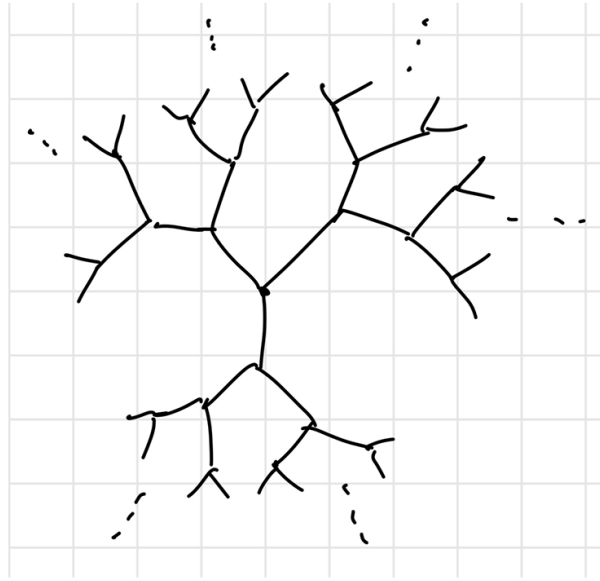


Figure 4: Resistance of tree network of identical resistors.

- i) When we pour hot water into an otherwise cold glass (cup), there is a chance the glass might unfortunately crack. Suppose, as a result of conduction of heat the temperature difference between the inner and outer surface of the glass is ΔT . The following are some information regarding the glass:

Young's modulus $E = 70 \text{ GPa}$

Coefficient of linear expansion $\alpha = 8.0 \times 10^{-6} \text{ K}^{-1}$

Compressive strength $\sigma = 21 \text{ MPa}$

- (i) By estimating the maximum stress developed in the glass, find the minimum temperature difference ΔT required to make the glass break.
- (ii) Is making the glass wall of the cup thicker a solution to this problem?

[5]

- j) Weights m_1 and m_2 hung from a massive chain with a mass of λ per unit length on a light frictionless pulley. The system is released from rest with the weights initial level, and then m_1 reaches the floor ($m_1 > m_2$). By considering the energy of the system, for what ratio of masses $\frac{m_1}{m_2}$ does the value of the mass of the chain not make any difference?

[4]

- k) Two tiny conducting spheres A and B of different masses are placed in a smooth hemispherical insulating bowl of radius 10 cm, and connected with an insulating spring with a natural length 8.0 cm and spring constant $k = 120 \text{ N m}^{-2}$, as illustrated in **Fig. 5**. The charges on A and B are $+2q$ and $+q$ respectively. At equilibrium, the arc length of A to the bottom of the bowl is $d_1 = 7.0 \text{ cm}$, and the arc length of B to the bottom of the bowl is $d_2 = 5.0 \text{ cm}$.

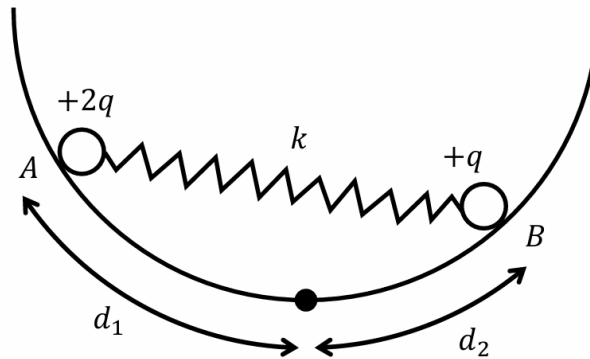


Figure 5: Two charges in a bowl

- (i) What is the elastic potential energy stored in the spring?
- (ii) Through a discharging process, the charge on A has been halved. This causes A to be displaced by 1.3 cm down along the bowl. How much will B be displaced along the bowl?

[6]

- 1) In an experiment, the count rate of a Geiger counter is the sum of a constant background rate B and that due to the radiation emitted directly from a certain decaying radioactive source. The total count rate was measured over a short time interval at the same time in three consecutive weeks. The rate measurements were C_1 , C_2 and C_3 respectively. Obtain an expression for B in terms of C_1 , C_2 and C_3 .

[4]

- m) Two weights, w_1 and w_2 are connected by a string which passes over a smooth pulley at the top of a smooth, inclined plane at 30° to the horizontal. One weight rest on the plane whilst the other hangs freely. They can be interchanged to compare their weight. w_1 draws w_2 up the plane in half the time that it takes for w_2 to draw up w_1 . What is the relation between the weights w_1 and w_2 ?

[5]

- n) Radiation is attenuated when it passes through material. The intensity decreases exponentially with distance from the surface of the material. A cuboid is made from a material which halves the radiation at a depth of 1.0 cm below the surface. When a beam of radiation is directed in turn along the three axes of the cuboid, the intensities are reduced to 60%, 70%, 80% of the incident. Determine the volume of the cuboid.

[4]

- o) An exploration ship has a circular glass filled circular porthole in the hull of the ship for studying the seabed. The porthole has a diameter of 40 cm which is much greater than the thickness of the glass. Calculate the area of seabed that is visible through the porthole if the refractive index of seawater is $n_w = 1.4$ and the seabed is 5.0 m below the porthole.

[4]

- p) The inner surface of a right circular cone is coated with a mirror-like reflecting material to form a conical mirror. A thin straight wire heated to white heat is stretched along the inner axis of the conical mirror. What is the minimum angle θ of the cone such that rays of light emitted by the filament will not be reflected off the cone more than once.

[4]

- q) The tractive force of a locomotive is plotted vertically with the speed horizontally in the graph of **Fig. 6**. Also shown on the same axes is the resistance of the train against its speed. If the weight of the train is 336 tonnes determine approximately the distance travelled before the train reaches its maximum speed.

1 tonne = 1000 kg

Tractive and Resistive forces acting on a train

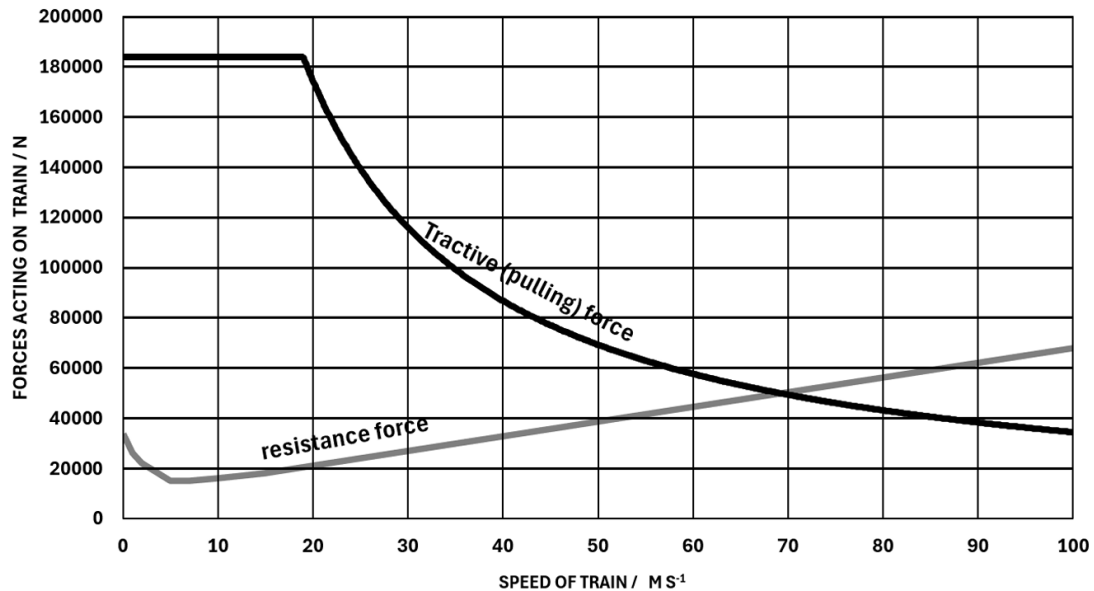


Figure 6: Graph showing the tractive and resistive forces acting on a train.

- r) As shown in **Fig. 7**, the ends of a massless, rigid rod of length L are attached to a set of rails which form a right angle to each other. The endpoints of the rod can slide freely without friction on the rails. A mass m is attached to the mid-point of the rod. Find the period T of small oscillations of the rod about its equilibrium position, and express your answers in terms of L , m and g .

[3]

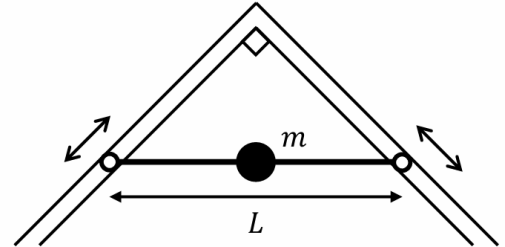


Figure 7: A massless, rigid rod attached to frictionless rails