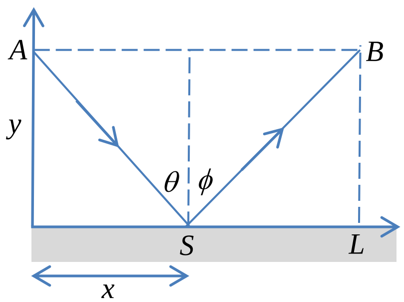
Reflection & Refraction

Consider a beam of light reflecting of a surface. What path does the light take? Feynman states that it takes all possible paths, although the probability of certain paths is more likely. Fermat's principle is more prescriptive – it states the path is the one which takes the *least time.*



Let the beam of light traverse a path ASB, reflecting off the surface at S.

The refractive index above the surface is assumed to be constant. The travel time is:

$$t=\frac{\sqrt{x^2+y^2}}{c/n}+\frac{\sqrt{(L-x)^2+y^2}}{c/n}$$

Variations in path can be characterized by changes in a single parameter, *x*. Hence we can determine the path of least travel time by finding a stationary value in the travel time as a function of *x*.

$$\frac{\partial t}{\partial x}=\frac{n}{c}\left(\frac{\frac{1}{2}2x}{\sqrt{x^2+y^2}}+\frac{\frac{1}{2}2(L-x)(-1)}{\sqrt{(L-x)^2+y^2}}\right)$$

The travel time is minimized when $\frac{\partial t}{\partial x}=0$

$$x-(L-x)=0$$

$$L=2x$$

$$\tfrac{1}{2}L=x$$

Since

$$x=y\tan\theta$$

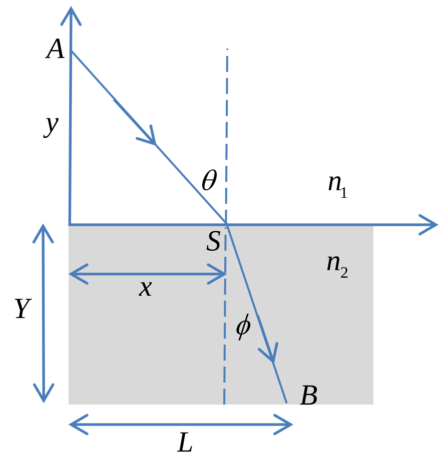
$$L-x=y\tan\phi$$

$$\therefore \tan\theta=\tan\phi$$

$$\therefore \boxed{\theta=\phi}$$

i.e. *the angle of incidence equals the angle of reflection*
This is the **Law of Reflection**

Now consider a similar situation for a ray refracted due to its passing through an interface between media of different refractive indices (i.e. where the speed of light varies)



$$t=\frac{\sqrt{x^2+y^2}}{c/n_1}+\frac{\sqrt{(L-x)^2+Y^2}}{c/n_2}$$

$$\frac{\partial t}{\partial x}=\frac{1}{c}\left(\frac{\frac{1}{2}2xn_1}{\sqrt{x^2+y^2}}+\frac{\frac{1}{2}2(L-x)(-1)n_2}{\sqrt{(L-x)^2+Y^2}}\right)$$

$$x=y\tan\theta$$

$$L-x=Y\tan\phi$$

$$\frac{\partial t}{\partial x}=\frac{1}{c}\left(\frac{y\tan\theta n_1}{y\sqrt{\tan^2\theta+1}}+\frac{-Y\tan\phi n_2}{Y\sqrt{\tan^2\phi+1}}\right)$$

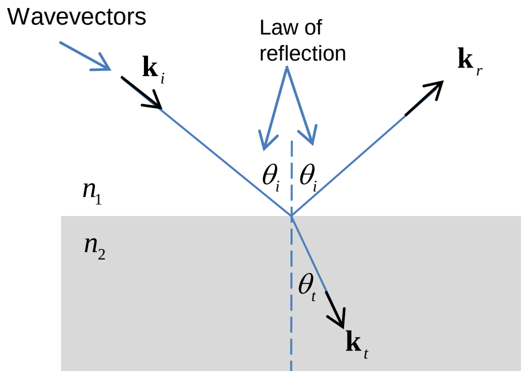
$$\frac{1}{\cos^2\theta}=1+\tan^2\theta \longrightarrow \frac{\partial t}{\partial x}=\frac{1}{c}\left(\cos\theta\tan\theta n_1-\cos\phi\tan\phi n_2\right)$$

$$\frac{\partial t}{\partial x}=\frac{1}{c}\left(n_1\sin\theta-n_2\sin\phi\right)$$

$$\therefore \frac{\partial t}{\partial x}=0\Rightarrow n_1\sin\theta=n_2\sin\phi$$

So the refracted ray path which minimizes the travel time is one which obeys *Snell's Law*

$$\boxed{n_1\sin\theta=n_2\sin\phi}$$



$$0 < \frac{n_1 \sin\theta_i}{n_2} < 1$$

Always true if $n_2 > n_1$

$n_1 > n_2$

$$\theta_i < \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Critical angle

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Snell's Law of refraction

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

$$\therefore \theta_t = \sin^{-1}\left(\frac{n_1 \sin\theta_i}{n_2}\right)$$

Real solutions for the transmitted ray angle when

$$0 < \frac{n_1 \sin\theta_i}{n_2} < 1$$

So EM waves propagating from a high refractive index medium to a lower refractive index medium will be **internally reflected** (i.e. no transmission) when the angle of incidence exceeds a ***critical angle*** θ_c