Hypothesis testing using the Binomial distribution

Random variable X is distributed by $\mathrm{B}(N,p)$. An observation x is made.
Null hypothesis H_0: $X \sim \mathrm{B}(N,p)$. Define a **significance level** s for *rejection* of H_0

In other words, B(10,0.25) is a valid hypothesis to explain the measurement of $x = 5$, assuming a Binomial distribution, and a 5% significance level.

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% ONE-TAILED TEST EXAMPLE
% Alternative H1: X ~ B(N,p*) where p*>p.
% Accept H1 and reject H0 if P(X>=x) < s.
% The critical region is the set of x values when P(X>=x) < s.

N=10; x=5; p=0.25; s = 0.05; test_type = 'p*>p';
Insufficient evidence to reject null hypothesis H0
X_crit = 6     7     8     9    10
p_value = 0.0781
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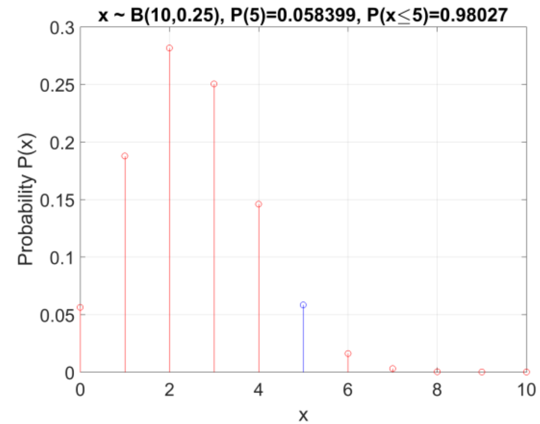

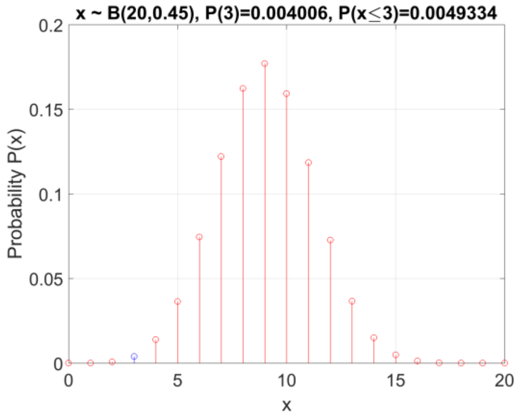


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% ONE-TAILED TEST EXAMPLE
% Alternative H2: X ~ B(N,p*) where p*<p.
% Accept H1 and reject H0 if P(X<=x) < s.
% The critical region is the set of X values when P(X<=x) < s

N=20; x=3; p=0.45; s = 0.01; test_type = 'p*<p';

Reject null hypothesis H0
X_crit = 0     1     2     3
p_value = 0.0049
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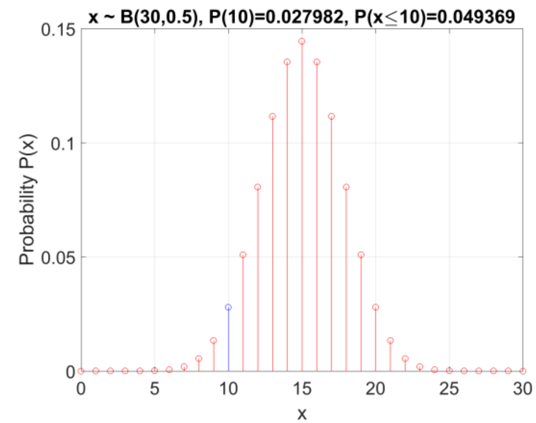
In other words, B(20,0.45) Is **NOT** a valid hypothesis to explain the measurement of $x = 3$, assuming A Binomial distribution, and a 1% significance level.



$$P(X \le x) = \sum_{k=0}^{x} \binom{N}{k} p^k (1-p)^{N-k}$$

$$P(X \ge x) = \sum_{x}^{N} \binom{N}{k} p^k (1-p)^{N-k}$$

Sum of Binomial probabilities

Binomial distribution

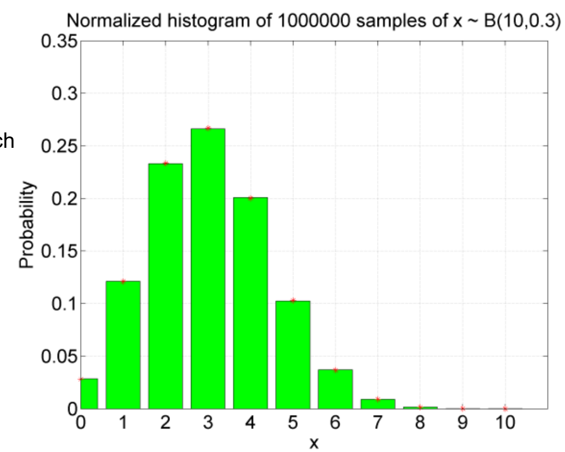
The random variable x is the number of successes out of n independent binary (success or failure) trials.
p is a fixed probability of success in each independent trial.

$x \sim \mathrm{B}(n,p)$

$$p(x,n,p) = \binom{n}{x} p^x (1-p)^{n-x}$$

$\mu = np$ mean

$\sigma^2 = np(1-p)$ variance

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% TWO-TAILED TEST EXAMPLE
% Random variable x is distributed by B(N,p). An observation X is made.
% Null hypothesis H0: x ~ B(N,p).  Alternative H1: x ~ B(N,p*) where p*~=p
% In this case, if X>N*p then consider a critical region when P(x>=X) < s/2.
% If X<N*p then consider a critical region when P(x<=X) < s/2.

N=30; x=10; p=0.5; s = 0.05; test_type = 'p*~=p';
Insufficient evidence to reject null hypothesis H0
X_crit = 0     1     2     3     4     5     6     7     8     9
p_value =0.0494
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