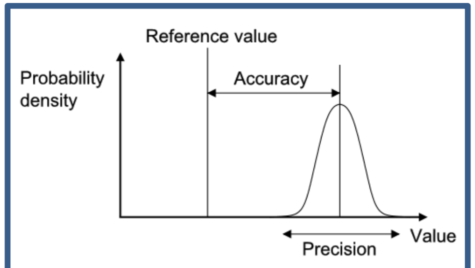
Measurement errors

When a measurement of a physical quantity (e.g. the length of an object) is made, the measurement will not be exact. Depending on the precision of the measuring device, there will always be a degree of uncertainty about what the measurement actually is.

Note *precision* is different from *accuracy*. An accurate instrument is one which is correctly *calibrated*, i.e. the mean measurement *aligns* which what is expected. A set of weighing scales which reads 0.1 kg unloaded is *both* inaccurate and imprecise compared to a laboratory standard weighing balance. If the scales are properly zeroed, then they are *only imprecise* compared to the lab equipment.



Note difference between *accuracy* and *precision*

If errors in precision can be thought to be *Gaussian*'(i.e. 'bell-curved' in shape) and the errors are *small* compared to the measurement taken, the ***Laws of Error Propagation*** can be used to work out errors in combined measurements, or in quantities formed by functions of measured quantities. (For the latter, the ***Laws of Error Propagation*** below assume the component measurements are *independent* or each other i.e. are *uncorrelated*).

Addition or subtraction

$$F = A \pm B$$
$$A = \bar{A} \pm \sigma_A$$
$$B = \bar{B} \pm \sigma_B$$
$$\bar{F} = \bar{A} \pm \bar{B}$$
$$\sigma_F = \sqrt{\sigma_A^2 + \sigma_B^2}$$
$$F = \bar{F} \pm \sigma_F$$

$$F = A \pm B$$
$$A = 100 \pm 3$$
$$B = 99 \pm 4$$

$$\bar{F} = 199$$
$$\sigma_F = \sqrt{3^2 + 4^2}$$

$$\therefore F = 199 \pm 5$$

Products

$$F = A^a B^b C^c .....$$
$$A = \bar{A} \pm \sigma_A$$
$$B = \bar{B} \pm \sigma_B$$
....
$$\bar{F} = \bar{A}^a \bar{B}^b \bar{C}^c .....$$
$$\sigma_F = \bar{F}\sqrt{\left(\frac{a\sigma_A}{\bar{A}}\right)^2 + \left(\frac{b\sigma_B}{\bar{B}}\right)^2 + \left(\frac{c\sigma_C}{\bar{C}}\right)^2}$$
$$F = \bar{F} \pm \sigma_F$$

$$F = \frac{A^2}{B}$$
$$A = 50 \pm 1$$
$$B = 100 \pm 2$$

$$\bar{F} = \frac{50^2}{100} = 25$$
$$\sigma_F = 25\sqrt{\left(\frac{2\times 1}{50}\right)^2 + \left(\frac{-1\times 2}{100}\right)^2} \approx 1.12$$

$$\therefore F = 25 \pm 1.12$$

For products (e.g. F = AB), think about *adding the squares of percentage errors*, then square rooting this.

Scaled addition or subtraction

$$F = aA \pm bB$$
$$A = \bar{A} \pm \sigma_A$$
$$B = \bar{B} \pm \sigma_B$$
$$\bar{F} = a\bar{A} \pm b\bar{B}$$
$$\sigma_F = \sqrt{a^2\sigma_A^2 + b^2\sigma_B^2}$$
$$F = \bar{F} \pm \sigma_F$$

$$F = 3A - \tfrac{1}{2}B$$
$$A = 100 \pm 3$$
$$B = 200 \pm 4$$

$$\bar{F} = 300 - 100 = 200$$
$$\sigma_F = \sqrt{3^2 \times 3^2 + \left(\tfrac{1}{2}\right)^2 \times 4^2} = \sqrt{82}$$

$$\therefore F = 200 \pm 9.1$$

General functions

$$F = f(A, B, C.....)$$
$$A = \bar{A} \pm \sigma_A$$
$$B = \bar{B} \pm \sigma_B$$
....
$$\bar{F} = f(\bar{A}, \bar{B}, \bar{C}.....)$$
$$\sigma_F = \sqrt{\left(\frac{\partial f}{\partial A}\right)^2 \sigma_A^2 + \left(\frac{\partial f}{\partial B}\right)^2 \sigma_B^2 + \left(\frac{\partial f}{\partial C}\right)^2 \sigma_C^2 + ...}$$
$$F = \bar{F} \pm \sigma_F$$

$$F = e^{2B} \ln|A|$$
$$A = \bar{A} \pm \sigma_A$$
$$B = \bar{B} \pm \sigma_B$$

$$\bar{F} = e^{2\bar{B}} \ln|\bar{A}|$$
$$\sigma_F = \sqrt{\left(\frac{e^{\bar{B}}}{\bar{A}}\right)^2 \sigma_A^2 + \left(2e^{2\bar{B}} \ln|\bar{A}|\right)^2 \sigma_B^2}$$
$$F = \bar{F} \pm \sigma_F$$