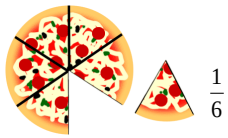
Fractions

Essentially a statement of *division* e.g. divide a pizza into six equal slices

$\frac{5}{6}$  $\frac{1}{6}$ "one sixth"

Multiplying fractions

$$\frac{5}{7}\times\frac{2}{3}=\frac{5\times2}{7\times3}=\frac{10}{21}$$

$$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$$

Reciprocals

$$1\div\frac{3}{5}=1/\tfrac{3}{5}=\left(\frac{3}{5}\right)^{-1}=\frac{5}{3}$$

$$1\div\frac{a}{b}=\frac{1}{a/b}=\frac{b}{a}$$

$$\left(\frac{a}{b}\right)^{-1}=\frac{b}{a}$$

Dividing fractions – same as multiplying by a reciprocal

$$\frac{3}{7}\div\frac{5}{11}=\frac{3}{7}\times\frac{11}{5}=\frac{33}{35}$$

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$$

Converting mixed fractions to top heavy fractions and vice versa

$$3\tfrac{5}{7}=3\times\frac{7}{7}+\frac{5}{7}=\frac{21+5}{7}=\frac{26}{7}$$

$$\frac{57}{5}=\frac{55+2}{5}=\frac{55}{5}+\frac{2}{5}=11\tfrac{2}{5}$$

Simplifying fractions. *Prime factorize*, then cancellation *may* occur!

$$\frac{14}{3}\times\frac{6}{7}=\frac{2\times7\times2\times3}{3\times7}=2\times2\times\frac{7}{7}\times\frac{3}{3}=\boxed{4}$$

Adding fractions. *Multiply by one* (e.g. 3/3 or 7/7) to *make the denominator the same*

$$\frac{2}{7}+\frac{1}{3}=\frac{2}{7}\times\boxed{\frac{3}{3}}+\frac{1}{3}\times\boxed{\frac{7}{7}}=\frac{6+7}{21}=\boxed{\frac{13}{21}}$$

$$\frac{a}{b}+\frac{c}{d}=\frac{a}{b}\times\boxed{\frac{d}{d}}+\frac{c}{d}\times\boxed{\frac{b}{b}}=\boxed{\frac{ad+cb}{bd}}$$

Subtracting fractions. Same method as adding fractions. Subtracting is essentially 'adding with negatives.'

$$\frac{2}{3}-\frac{1}{5}=\frac{2}{3}\times\boxed{\frac{5}{5}}-\frac{1}{5}\times\boxed{\frac{3}{3}}=\frac{10-3}{15}=\boxed{\frac{7}{15}}$$

$$\frac{a}{b}-\frac{c}{d}=\frac{a}{b}\times\boxed{\frac{d}{d}}-\frac{c}{d}\times\boxed{\frac{b}{b}}=\boxed{\frac{ad-cb}{bd}}$$

Converting a recurring decimal into a fraction

$x=0.\dot{3}=0.33333333....$

$10x=3.\dot{3}$

$\therefore 10x-x=3.\dot{3}-0.\dot{3}$

$9x=3$

$\therefore x=\boxed{\frac{1}{3}}$

$x=6.\dot{7}=7.77777777....$

$10x=67.\dot{7}$

$\therefore 10x-x=67.\dot{7}-6.\dot{7}$

$9x=61$

$\therefore x=\frac{61}{9}=\frac{54+7}{9}=\boxed{6\frac{7}{9}}$

$x=1.2\dot{3}\dot{3}=1.2323232323$

$100x=123.2\dot{3}\dot{3}$

$\therefore 100x-x=123.2\dot{3}\dot{3}-1.2\dot{3}\dot{3}$

$99x=122$

$\therefore x=\frac{122}{99}=\boxed{1\frac{23}{99}}$

$x=0.456745674567=0.\dot{4}\dot{5}\dot{6}\dot{7}$

$10,000x=4567.\dot{4}\dot{5}\dot{6}\dot{7}$

$9,999x=4567$

$\therefore x=\boxed{\frac{4567}{9999}}$

Numerator → n

Denominator → d

Continued fractions

$$\frac{1}{1+\frac{1}{1+\frac{1}{2}}}=\frac{1}{1+\frac{1}{\frac{3}{2}}}=\frac{1}{1+\frac{2}{3}}=\frac{1}{\frac{3}{3}+\frac{2}{3}}=\frac{1}{\frac{5}{3}}=\frac{3}{5}$$

Golden ratio

$$\phi=\frac{1+\sqrt{5}}{2}=1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+.....}}}}}$$

$$\phi=1+\frac{1}{\phi}$$

$$\phi^2-\phi-1=0$$

$$\sqrt{2}=1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+.....}}}$$

$$\sqrt{2}=1+\frac{1}{\sqrt{2}+1}$$

$$\sqrt{2}=\frac{\sqrt{2}+1+1}{\sqrt{2}+1}=\frac{\sqrt{2}+2}{\sqrt{2}+1}=\frac{\sqrt{2}\left(1+\sqrt{2}\right)}{\sqrt{2}+1}=\sqrt{2}$$

$$\pi=3+\cfrac{1^2}{6+\cfrac{3^2}{6+\cfrac{5^2}{6+\cfrac{7^2}{6+\cfrac{9^2}{6+.....}}}}}$$