Newton's Law of Cooling

Consider a vessel containing a fluid of uniform temperature. The walls of the vessel are of uniform thickness and enclose the fluid. The fluid is at temperature T and the ambient temperature outside the vessel is T_a

If one assumes heat transfer between the fluid and surroundings (e.g. air) is via **conduction** (i.e. not *radiative transfer* or via physical transfer of fluid particles from the vessel to surroundings, as in *evaporative cooling*) then we can use **Fourier's Law** to determine the heat flux between the vessel and the surroundings – i.e. the *rate of heat transferred per unit area of the vessel is proportional to the temperature gradient between the fluid and the surroundings*.

Q is the heat transferred from the vessel to the surroundings
k is the thermal conductivity of the vessel
Δx is the thickness of the vessel
A is the surface area of the vessel

$$\frac{dQ}{dt} = kA\frac{(T - T_a)}{\Delta x}$$

If the specific heat capacity of the fluid is c, and the vessel contains m kg of fluid

$$dQ = -mcdT$$

If we assume the heat capacity is independent of temperature*

$$c \approx \frac{3R}{M}$$

Molar heat capacity ($3R$)

Molar mass /kg (M)

$$\frac{dT}{dt} = -\frac{kA}{mc\Delta x}(T - T_a)$$

$$\int_{T_0}^{T} \frac{dT}{T - T_a} = -\frac{kA}{mc\Delta x}\int_0^t dt$$

$$\left[\ln|T - T_a|\right]_{T_0}^{T} = -\frac{kAt}{mc\Delta x}$$

Let us assume the initial fluid temperature $T_0 > T_a$

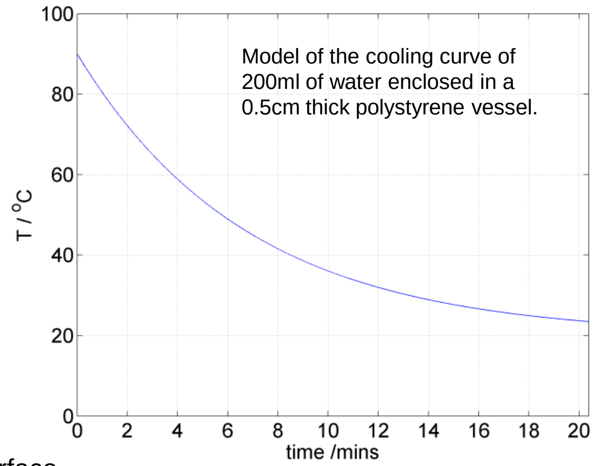
$$\ln\left(\frac{T - T_a}{T_0 - T_a}\right) = -\frac{kAt}{mc\Delta x}$$

$$\frac{T - T_a}{T_0 - T_a} = e^{-\frac{kAt}{mc\Delta x}}$$

$$T = T_a + (T_0 - T_a)e^{-\frac{kAt}{mc\Delta x}}$$

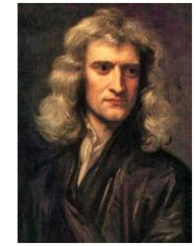
So temperature of the fluid inside the vessel *decays exponentially*, with a decay rate which depends upon the thermal conductivity of the vessel, the thickness of the walls, the surface area, the mass of fluid and the heat capacity of the fluid.

This is **Newton's Law of Cooling**



Note a sensible cooling time constant is

$$\tau = \frac{mc\Delta x}{kA}$$



Isaac Newton
(1643-1727)

Joseph Fourier
(1768-1830)

Material	k /Js^{-1}m^{-1}K^{-1}	c /Jkg^{-1}K^{-1}
Silica aerogel	0.02	840
Air	0.02	1,012
Paper	0.05	1,336
Dry snow	0.1	2,090
Rubber	0.2	2,010
Polystyrene	0.3	1,300
Brick	0.5	840
Water	0.6	4,181
Glass	1	500-840
Concrete	1.5	880
Ice	2	2,110

Material	k /Js^{-1}m^{-1}K^{-1}	c /Jkg^{-1}K^{-1}
Graphene	5000	1,000?
Copper	380	385
Aluminium	220	870
Iron	55	450
Solder	50	180

Table of thermal conductivities and typical specific heat capacities of various materials at around room temperature.

*This is not a good model for low temperatures, but at room temperature, and higher, most liquids and solids have a molar heat capacity of about $3R$, where $R = 8.314$Jmol^{-1}K^{-1}, the molar gas constant. This called the **Dulong-Petit law**, and is based upon the idea of thermal energy in a solid arsing from atomic lattice vibrations, which are constrained to have discrete energies by the quantum nature of these oscillators.