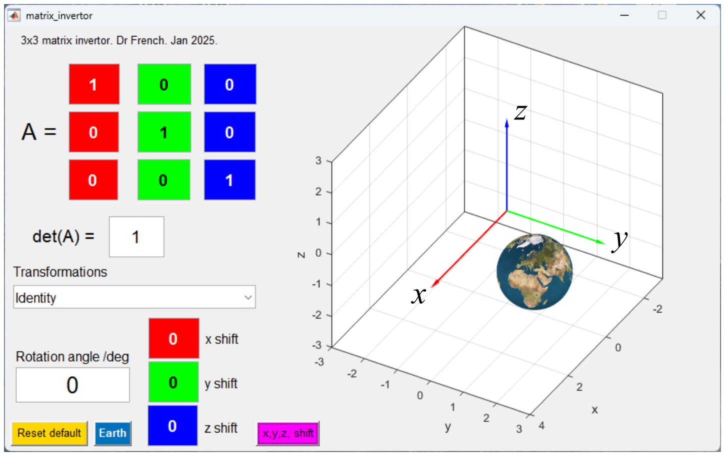
Multiplication by a 3x3 matrix can apply a **geometric transformation about the origin** to a 3xN array of (x,y,z) coordinates, which represent some form of surface or 3D structure.

This is very useful in computer aided design (CAD), in fact any form of digital rendering (movies, TV, games …)

$$\begin{pmatrix} x_1' & x_2' & \\ y_1' & y_2' & \dots \\ z_1' & z_2' & \end{pmatrix} = \mathbf{A} \begin{pmatrix} x_1 & x_2 & \\ y_1 & y_2 & \dots \\ z_1 & z_2 & \end{pmatrix}$$

As for 2D geometric transformations, the 3D transformation matrices can be determined by what happens to the **basis vectors** under the transformation.

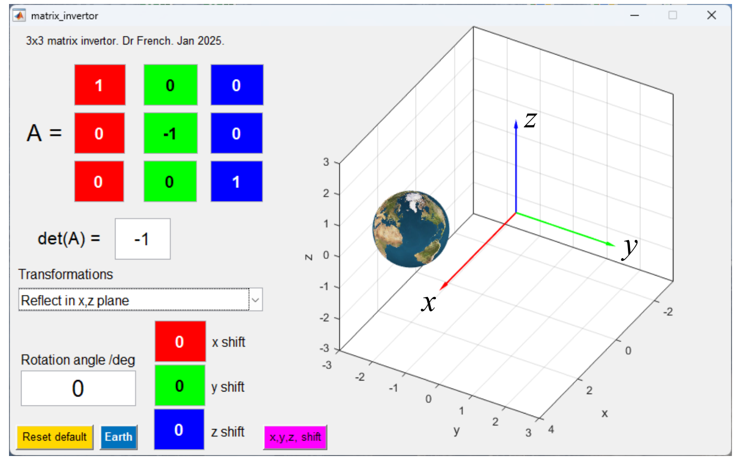
$$\underset{\hat{\mathbf{x}}}{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}} \to \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad \underset{\hat{\mathbf{y}}}{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}} \to \begin{pmatrix} d \\ e \\ f \end{pmatrix} \quad \underset{\hat{\mathbf{z}}}{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}} \to \begin{pmatrix} g \\ h \\ i \end{pmatrix} \quad \therefore \mathbf{A} = \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix}$$



$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

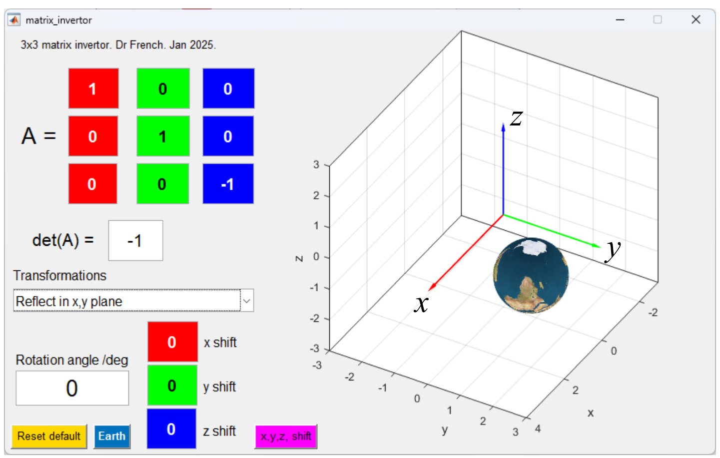
Identity

i.e. no change!



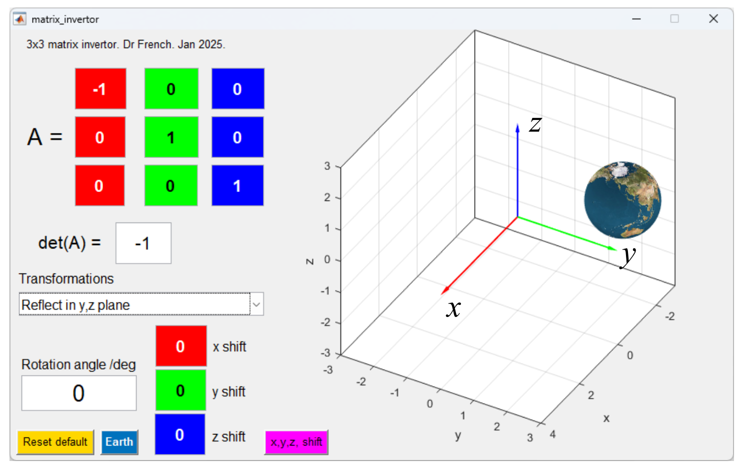
$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Reflection in x,z plane



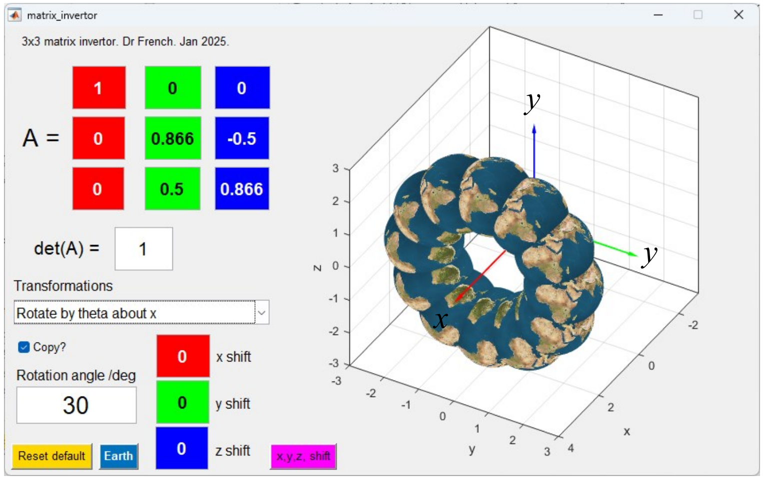
$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Reflection in x,y plane



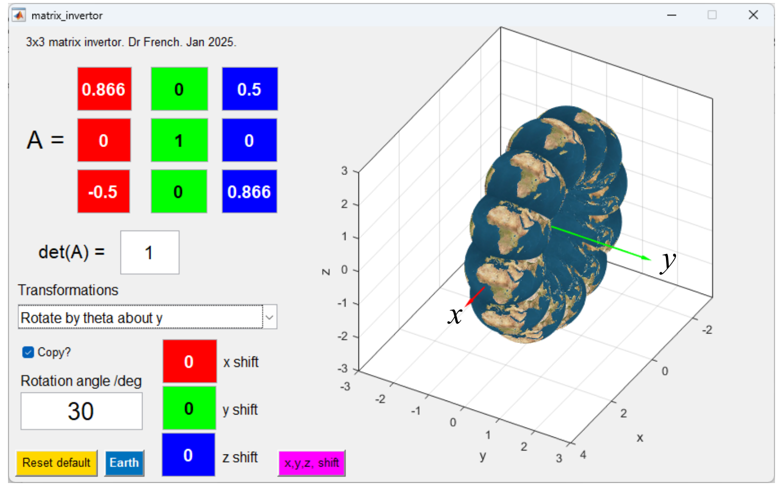
$$\mathbf{A} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Reflection in y,z plane



$$\mathbf{A}=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\theta & -\sin\theta\\ 0 & \sin\theta & \cos\theta\end{pmatrix}$$

Rotation by θ anticlockwise about the x axis

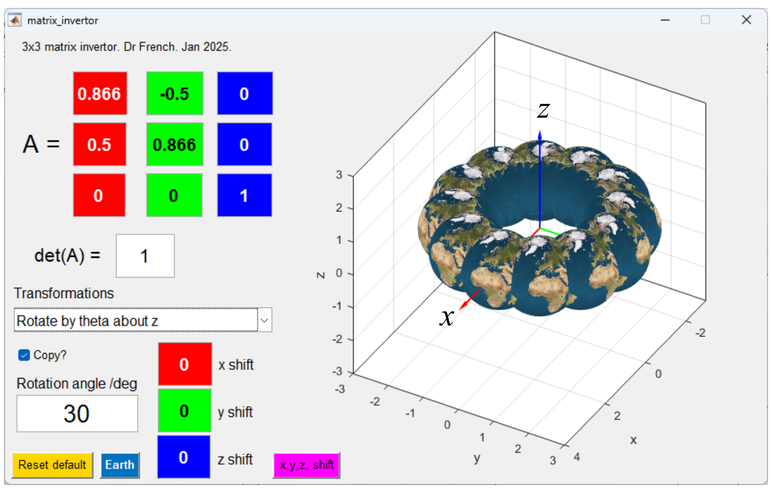


$$\mathbf{A}=\begin{pmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{pmatrix}$$

Rotation by θ anticlockwise about the y axis

$$\mathbf{A}=\begin{pmatrix}k & 0 & 0\\ 0 & k & 0\\ 0 & 0 & k\end{pmatrix}$$

Enlargement by factor k from the origin.



$$\mathbf{A}=\begin{pmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{pmatrix}$$

Rotation by θ anticlockwise about the z axis