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BPhO Round 1

Section 1

11^{th} November 2022

This question paper must not be taken out of the exam room

Instructions

Time: **1 hour 20 minutes** for this section.

Questions: Students may attempt any parts of *Section 1*, but are not expected to complete all parts.

Working: Working, calculations, explanations and **diagrams**, properly laid out, must be shown for full credit. The final answer alone is not sufficient. Writing must be clear.
Marks are given for intermediate steps if they can be seen: underline or circle them so that the marker can find them.

Marks: A **maximum of 50 marks** can be awarded for *Section 1*. There is a total of $\approx$ **84** marks allocated to the problems of Question 1 which makes up the whole of *Section 1*.

Instructions: You are allowed any standard exam board data/formula sheet.

Calculators: Any standard calculator may be used, but calculators must not have symbolic algebra capability. If they are programmable, then they must be cleared or used in "exam mode". Code may not be written for any of the BPhO competitions.

Solutions: 1. Answers and calculations are to be written on loose paper **ON ONE SIDE ONLY (pages will be scanned)**. 2. Students should write their **name** and their **school/college** clearly on every answer sheet. 3. Number each question clearly. 4. Number your pages at the top. 5. Write "END" at the end of your script. 6. Fill in the Front Cover Sheet your teacher will give you - **just one for the two sections**.

Sitting the paper: There are two options for sitting BPhO Round 1:

a. *Section 1* and *Section 2* may be sat in one session of 2 hours 40 minutes. ***Section 1* should be collected in after 1 hour 20 minutes** and only then should *Section 2* be given out.

b. *Section 1* and *Section 2* may be sat in two separate sessions of 1 hour 20 minutes each. *Section 1* must be collected in after the first session and *Section 2* only handed out at the beginning of the second session.

Important Constants

Constant	Symbol	Value
Speed of light in free space	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Elementary charge	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Planck constant	h	$6.63 \times 10^{-34}\,\mathrm{J\,s}$
Mass of electron	m_{e}	$9.110 \times 10^{-31}\,\mathrm{kg}$
Mass of proton	m_{p}	$1.673 \times 10^{-27}\,\mathrm{kg}$
Mass of neutron	m_{p}	$1.675 \times 10^{-27}\,\mathrm{kg}$
atomic mass unit	u	$1.661 \times 10^{-27}\,\mathrm{kg} = 931.5\,\mathrm{MeV\,c^{-2}}$
Gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{m^{3}\,kg^{-1}\,s^{-2}}$
Earth's gravitational field strength	g	$9.81\,\mathrm{N\,kg^{-1}}$
Permittivity of free space	ε_0	$8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Avogadro constant	N_{A}	$6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Gas constant	R	$8.3145\,\mathrm{J\,K^{-1}\,mol^{-1}}$
Mass of Sun	M_{S}	$1.99 \times 10^{30}\,\mathrm{kg}$
Radius of Earth	R_{E}	$6.37 \times 10^{6}\,\mathrm{m}$
Specific heat capacity of water	c_{w}	$4180\,\mathrm{J\,kg^{-1}\,{}^{\circ}C^{-1}}$

$$T_{(\mathrm{K})} = T_{(^{\circ}\mathrm{C})} + 273$$

$$\text{Volume of a sphere} = \tfrac{4}{3}\pi r^{3}$$

$$e^{x} \approx 1 + x + \ldots \qquad \text{for } x \ll 1$$

$$(1+x)^{n} \approx 1 + nx \qquad \text{for } x \ll 1$$

$$\frac{1}{(1+x)^{n}} \approx 1 - nx \qquad \text{for } x \ll 1$$

$$\tan\theta \approx \sin\theta \approx \theta \qquad \text{for } \theta \ll 1$$

$$\cos\theta \approx 1 - \frac{\theta^{2}}{2} \qquad \text{for } \theta \ll 1$$

Section 1 — 50 marks maximum

Question 1

a) A steel ball is thrown down with a speed of $3.0\,\mathrm{m\,s^{-1}}$ on to a hard surface from a height of $2.0\,\mathrm{m}$. It retains 70% of its energy on each bounce. Calculate

(i) the speed at which it hits the ground for the first time, and

(ii) the maximum height it reaches after the 4^{th} bounce.

[2]

b) A long-distance cyclist uses a cycle computer that is dual-powered: it has an internal battery and a solar panel. While the cyclist is riding in direct sunlight, the solar panel on the computer provides energy at a rate equal to $\frac{1}{3}$ of the power consumption. If a fully-charged cycle computer lasts 10 hours when riding at night, calculate how long a fully charged computer will run for in direct sunlight.

[2]

c) Riding round a corner at $10\,\mathrm{km\,h^{-1}}$ a cyclist leans over at an angle of $12°$ to the vertical. At what angle would they lean over if they went round the corner at $15\,\mathrm{km\,h^{-1}}$?

[3]

d) Io and Europa are both moons of Jupiter. Europa takes twice as long as Io to complete an orbit. What is the ratio of the centripetal acceleration of Io and Europa, $\dfrac{a_{\text{Io}}}{a_{\text{Europa}}}$?

You may use the result that for this gravitational system, $\omega^2 r^3 = \text{constant}$, where ω is the angular velocity and r is the radius of the orbit.

[2]

e) A particle of mass m_1 and initial speed u makes an elastic collision with a stationary particle of mass m_2. The particles move off at speeds v_1 and v_2 respectively, at equal angles θ either side of the initial incident direction of m_1.

(i) What is the largest ratio of $\dfrac{m_1}{m_2}$ for which this equal angle condition can occur?

(ii) If $m_1 = m_2$, what is the largest angle of deflection, θ, of particle m_1 for this equal angle condition?

[5]

f) A body is projected with velocity v up a plane inclined at an angle α to the horizontal. When it returns through its starting point it is moving with half the speed with which is was projected.
Determine the coefficient of friction μ, in terms of the angle of the plane.

Hint: The coefficient of friction is given by $F_{\text{friction}} = \mu N$ where N is the normal contact force and F_{friction} is the frictional force on the body.

[4]

g) A cylindrical container is filled with equal volumes of n different liquids which do not mix, so that they form horizontal layers each of height h. The densities of the liquids are ρ, 2ρ, $3\rho, \cdots$, with the lowest liquid of density $n\rho$. The curved surface area of the cylinder enclosing each liquid is A.

(i) Give an expression for the force F_1 on the area A surrounding the top liquid in terms of ρ, g, h, A.

(ii) What is the force F_2 on the surface A surrounding the second liquid down from the top, in terms of F_1?

(iii) Give an expression for the force F_n on the area A surrounding the n^{th} liquid at the bottom of the cylinder in term of F_1?

$$\text{Hint :} \quad 1+2+3+\cdots+n = \sum_{i=1}^{n} i = \frac{n}{2}(n+1)$$

[4]

h) A rocket of mass $m_{\mathrm{r}} = 5000\,\mathrm{kg}$ contains a further mass $m_0 = 5000\,\mathrm{kg}$ of fuel. Once the fuel is ignited, 50 kg per second of hot gas is expelled downwards at a speed of $2000\,\mathrm{m\,s^{-1}}$.

(i) Calculate the thrust, T, applied to the rocket,

(ii) Find an expression for the acceleration of the rocket, a, in terms of its total mass m, T and g,

(iii) Find an expression for the acceleration of the rocket as a function of time, t, in terms of $T, g, m_{\mathrm{r}}, m_0$ and t_0, the total time for which the thrust acts.

(iv) Calculate the time after launch at which the weight of an astronaut on board will have appeared to double.

[4]

i) An open topped steel drum is completely filled with oil on a day when the temperature is $5.0\,^{\circ}\mathrm{C}$. On a warm day the temperature rises, and 2.4% of the oil spills out.
What is the temperature reached on that day?

The **volume coefficient** of expansion of oil is $7.0 \times 10^{-4}\,^{\circ}\mathrm{C}^{-1}$
The **linear coefficient** of expansion of steel is $1.2 \times 10^{-5}\,^{\circ}\mathrm{C}^{-1}$

[3]

j) Observed from an air traffic control tower, an aeroplane has a bearing of 068° and a range of 43 km. Five minutes later the bearing of the aircraft is 040° with a range of 52 km. Determine

(i) The speed of the aircraft in $\mathrm{m\,s^{-1}}$.

(ii) Its bearing and range 10 minutes after the second sighting.

[4]

k) A gas is found to obey the equation relating p, V, n, R, T

$$p(V - b) = nRT\exp\left(\frac{-a}{nRTV}\right)$$

where p is the gas pressure
V is its volume
R is the molar gas constant
n is the number of moles
a and b are constants.

(i) Determine the SI base units (m, kg, s) in which a and b are expressed.

(ii) If $b \ll V$ and $a \ll nRTV$, show that this expression approximates to the ideal gas equation relating p, V, n, R, T at a particular temperature T_c. Determine T_c in terms of a, b, n and R.

Hint: For $x \ll 1$ $\quad \exp(x) = e^x \approx 1 + x$

[4]

l) A thin rod is balanced in a smooth hemispherical bowl fixed to a table, touching both the interior and the rim as shown in **Fig. 1**. The rim of the bowl remains horizontal. Expressed in the simplest form, determine the radius of the bowl r in terms of the length of the rod ℓ, and the angle θ to the horizontal.

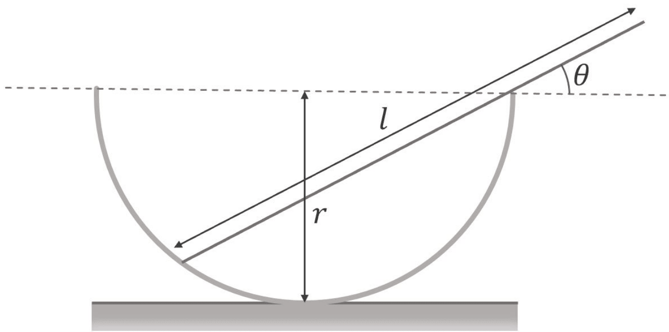

Figure 1: A rod in a smooth bowl.

[6]

m) In the circuit shown in **Fig. 2**, the three cells each supply an emf of 5.0 V and have an internal resistance of 5.0 Ω. The external resistors also each have a resistance of 5.0 Ω.
What arrangement of switches gives

(i) the maximum current,

(ii) the minimum non-zero current.

(iii) Determine the current in each case.

[3]

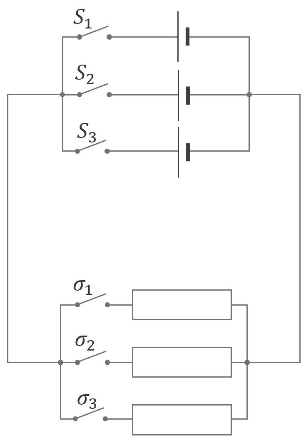

Figure 2: Three cells and switches in parallel.

n) The circuit of **Fig. 3** consists of four resistors and a switch **S**. When the **S** is open, the current flowing through the 5 kΩ resistor is I_o. When **S** is closed, the current flowing through the same resistor is I_c. What is the ratio $\frac{I_c}{I_o}$, giving your answer as the ratio of two integers.

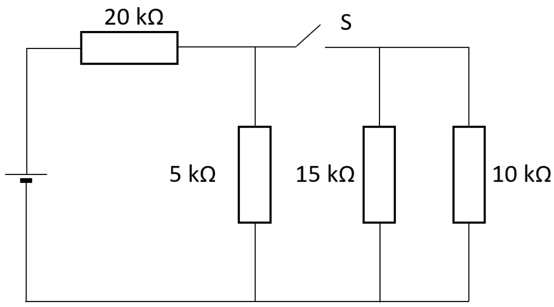

Figure 3: A switched circuit.

[4]

o) The Stefan-Boltzmann law says that the emitted power of a spherical "black body", P, is related to the radius, R, and absolute surface temperature, T, as $P \propto R^2T^4$. Wien's "displacement law" says that the wavelength λ_{max} corresponding to the peak value of the emitted power of this spectrum is inversely proportional to the absolute surface temperature. The Sun currently has its peak wavelength as 500 nm.
What will be the new peak wavelength when it becomes a red giant, given its radius will be 200 times larger and its power output 4000 times larger?

[3]

p) One spectral line in hydrogen is caused by photons with an energy of 2.55 eV. The same line is redshifted in the spectrum of a distant galaxy by 5.4 nm. Calculate

(i) the wavelength of the photon,

(ii) the speed of recession of the galaxy,

(iii) the distance to the galaxy.

How far away is the galaxy? Give your answer in megaparsecs (Mpc).

The Hubble constant, $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

[3]

q) A car drives along a road that has small depressions regularly spaced about 8.0 m apart. When four 80 kg passengers enter the 800 kg car, it sinks down by 1.8 cm.
At approximately what speed might travelling in the vehicle become very uncomfortable?

[3]

r) A pendulum clock is controlled by the swing of a simple pendulum (a mass on the end of a light rod) and is intended to have a period of 1.00 seconds. However, the clock runs slow by ten minutes each day. What percentage change should be made in the length of the pendulum?

[3]

s) A binary star system is 2140 light years away and consists of two stars like the Sun. The average separation between the stars is 0.00593 light years.
Determine the diameter of the telescope needed to resolve them if using a visible wavelength of 550 nm.

[2]

t) A glass prism of refracting angle 75.0° is shown in **Fig. 4** has a refractive index of $n = 1.40$.

(i) For what range of incident angles will light from air that is incident on face **AB** emerge from face **AC**?

(ii) Show your result on a diagram.

[3]

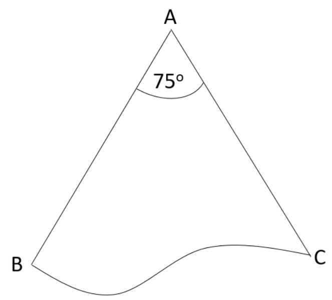

Figure 4: Glass prism with an apex angle of 75°.

u) A particle **A**, of mass m carrying a charge of Q is suspended by an insulating thread of length ℓ. Another particle **B**, of negligible mass but of positive charge $+q$ is brought towards **A**, which is repelled. When **B** arrives at the point previously occupied by **A**, the system is in (neutral) equilibrium.
Calculate the work done in terms of m, g, ℓ, q, Q, and k, where $k = \dfrac{1}{4\pi\epsilon_0}$.

[6]

v) A capacitor of value 1.0 F discharges through a device whose resistance R varies linearly with applied potential difference, V, so that $R = AV + B$, where A and B are constants. The resistance of the device has a value of 10.0 Ω when $V = 6.0$ V, and 4.0 Ω when $V = 0.06$ V.
The capacitor is initially charged to a potential of 6.0 V.
Determine how long it takes for the capacitor to discharge to 1% of this initial value.

[6]

w) The $^{238}_{92}\mathrm{U}$ decays according to

$$^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$$

Determine the kinetic energy of the emitted α−particle in MeV.

$c = 2.998 \times 10^{8}\ \mathrm{m\,s^{-1}}$
Mass of the $^{238}_{92}\mathrm{U}$ nucleus is 3.85395×10^{-25} kg
Mass of the $^{234}_{90}\mathrm{Th}$ nucleus is 3.78737×10^{-25} kg
Mass of the α−particle is 6.64807×10^{-27} kg

[5]

END OF SECTION 1