

- a) A steel ball is thrown down with a speed of  $3.0 \text{ m s}^{-1}$  on to a hard surface from a height of  $2.0 \text{ m}$ . It retains  $70\%$  of its energy on each bounce. Calculate
- (i) the speed at which it hits the ground for the first time, and
  - (ii) the maximum height it reaches after the 4<sup>th</sup> bounce.

[2]

b) A long-distance cyclist uses a cycle computer that is dual-powered: it has an internal battery and a solar panel. While the cyclist is riding in direct sunlight, the solar panel on the computer provides energy at a rate equal to  $\frac{1}{3}$  of the power consumption. If a fully-charged cycle computer lasts 10 hours when riding at night, calculate how long a fully charged computer will run for in direct sunlight.

[2]

c) Riding round a corner at  $10 \text{ km h}^{-1}$  a cyclist leans over at an angle of  $12^\circ$  to the vertical.  
At what angle would they lean over if they went round the corner at  $15 \text{ km h}^{-1}$ ?

**[3]**

d) Io and Europa are both moons of Jupiter. Europa takes twice as long as Io to complete an orbit. What is the ratio of the centripetal acceleration of Io and Europa,  $\frac{a_{\text{Io}}}{a_{\text{Europa}}}$ ?

You may use the result that for this gravitational system,  $\omega^2 r^3 = \text{constant}$ , where  $\omega$  is the angular velocity and  $r$  is the radius of the orbit.

[2]

e) A particle of mass  $m_1$  and initial speed  $u$  makes an elastic collision with a stationary particle of mass  $m_2$ . The particles move off at speeds  $v_1$  and  $v_2$  respectively, at equal angles  $\theta$  either side of the initial incident direction of  $m_1$ .

- (i) What is the largest ratio of  $\frac{m_1}{m_2}$  for which this equal angle condition can occur?
- (ii) If  $m_1 = m_2$ , what is the largest angle of deflection,  $\theta$ , of particle  $m_1$  for this equal angle condition?

[5]

- f) A body is projected with velocity  $v$  up a plane inclined at an angle  $\alpha$  to the horizontal. When it returns through its starting point it is moving with half the speed with which it was projected.

Determine the coefficient of friction  $\mu$ , in terms of the angle of the plane.

Hint: The coefficient of friction is given by  $F_{\text{friction}} = \mu N$  where  $N$  is the normal contact force and  $F_{\text{friction}}$  is the frictional force on the body.

[4]

g) A cylindrical container is filled with equal volumes of  $n$  different liquids which do not mix, so that they form horizontal layers each of height  $h$ . The densities of the liquids are  $\rho, 2\rho, 3\rho, \dots$ , with the lowest liquid of density  $n\rho$ . The curved surface area of the cylinder enclosing each liquid is  $A$ .

- (i) Give an expression for the force  $F_1$  on the area  $A$  surrounding the top liquid in terms of  $\rho, g, h, A$ .
- (ii) What is the force  $F_2$  on the surface  $A$  surrounding the second liquid down from the top, in terms of  $F_1$ ?
- (iii) Give an expression for the force  $F_n$  on the area  $A$  surrounding the  $n^{\text{th}}$  liquid at the bottom of the cylinder in term of  $F_1$ ?

Hint :  $1 + 2 + 3 + \dots + n = \sum_{i=1}^n i = \frac{n}{2}(n + 1)$

[4]

- h) A rocket of mass  $m_r = 5000$  kg contains a further mass  $m_0 = 5000$  kg of fuel. Once the fuel is ignited, 50 kg per second of hot gas is expelled downwards at a speed of  $2000 \text{ m s}^{-1}$ .
- (i) Calculate the thrust,  $T$ , applied to the rocket,
  - (ii) Find an expression for the acceleration of the rocket,  $a$ , in terms of its total mass  $m$ ,  $T$  and  $g$ ,
  - (iii) Find an expression for the acceleration of the rocket as a function of time,  $t$ , in terms of  $T$ ,  $g$ ,  $m_r$ ,  $m_0$  and  $t_0$ , the total time for which the thrust acts.
  - (iv) Calculate the time after launch at which the weight of an astronaut on board will have appeared to double.

[4]



- i) An open topped steel drum is completely filled with oil on a day when the temperature is  $5.0^{\circ}\text{C}$ . On a warm day the temperature rises, and 2.4% of the oil spills out. What is the temperature reached on that day?

The **volume coefficient** of expansion of oil is  $7.0 \times 10^{-4} \text{ }^{\circ}\text{C}^{-1}$

The **linear coefficient** of expansion of steel is  $1.2 \times 10^{-5} \text{ }^{\circ}\text{C}^{-1}$

[3]

j) Observed from an air traffic control tower, an aeroplane has a bearing of  $068^\circ$  and a range of 43 km. Five minutes later the bearing of the aircraft is  $040^\circ$  with a range of 52 km. Determine

(i) The speed of the aircraft in  $\text{m s}^{-1}$ .

(ii) Its bearing and range 10 minutes after the second sighting.

**[4]**

k) A gas is found to obey the equation relating  $p, V, n, R, T$

$$p(V - b) = nRT \exp\left(\frac{-a}{nRTV}\right)$$

where  $p$  is the gas pressure

$V$  is its volume

$R$  is the molar gas constant

$n$  is the number of moles

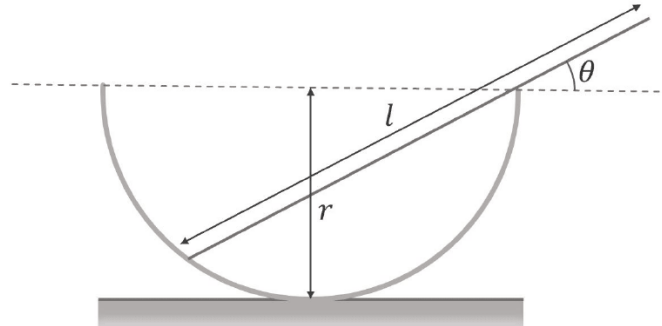
$a$  and  $b$  are constants.

- (i) Determine the SI base units (m, kg, s) in which  $a$  and  $b$  are expressed.
- (ii) If  $b \ll V$  and  $a \ll nRTV$ , show that this expression approximates to the ideal gas equation relating  $p, V, n, R, T$  at a particular temperature  $T_c$ . Determine  $T_c$  in terms of  $a, b, n$  and  $R$ .

Hint: For  $x \ll 1$       $\exp(x) = e^x \approx 1 + x$

[4]

- 1) A thin rod is balanced in a smooth hemispherical bowl fixed to a table, touching both the interior and the rim as shown in **Fig. 1**. The rim of the bowl remains horizontal. Expressed in the simplest form, determine the radius of the bowl  $r$  in terms of the length of the rod  $\ell$ , and the angle  $\theta$  to the horizontal.



**Figure 1:** A rod in a smooth bowl.

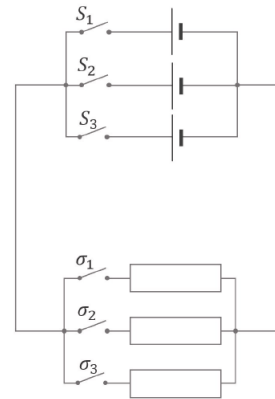
[6]

m) In the circuit shown in **Fig. 2**, the three cells each supply an emf of 5.0 V and have an internal resistance of 5.0  $\Omega$ . The external resistors also each have a resistance of 5.0  $\Omega$ .

What arrangement of switches gives

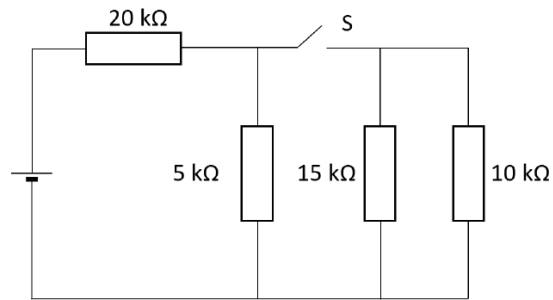
- (i) the maximum current,
- (ii) the minimum non-zero current.
- (iii) Determine the current in each case.

[3]



**Figure 2:** Three cells and switches in parallel.

- n) The circuit of **Fig. 3** consists of four resistors and a switch **S**. When the **S** is open, the current flowing through the  $5\text{ k}\Omega$  resistor is  $I_0$ . When **S** is closed, the current flowing through the same resistor is  $I_c$ . What is the ratio  $\frac{I_c}{I_0}$ , giving your answer as the ratio of two integers.



**Figure 3:** A switched circuit.

[4]

- o) The Stefan-Boltzmann law says that the emitted power of a spherical “black body”,  $P$ , is related to the radius,  $R$ , and absolute surface temperature,  $T$ , as  $P \propto R^2 T^4$ . Wien’s “displacement law” says that the wavelength  $\lambda_{\max}$  corresponding to the peak value of the emitted power of this spectrum is inversely proportional to the absolute surface temperature. The Sun currently has its peak wavelength as 500 nm. What will be the new peak wavelength when it becomes a red giant, given its radius will be 200 times larger and its power output 4000 times larger?

[3]

p) One spectral line in hydrogen is caused by photons with an energy of 2.55 eV. The same line is redshifted in the spectrum of a distant galaxy by 5.4 nm. Calculate

- (i) the wavelength of the photon,
- (ii) the speed of recession of the galaxy,
- (iii) the distance to the galaxy.

How far away is the galaxy? Give your answer in megaparsecs (Mpc).

The Hubble constant,  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ .

**[3]**



- q) A car drives along a road that has small depressions regularly spaced about 8.0 m apart. When four 80 kg passengers enter the 800 kg car, it sinks down by 1.8 cm. At approximately what speed might travelling in the vehicle become very uncomfortable?

**[3]**

r) A pendulum clock is controlled by the swing of a simple pendulum (a mass on the end of a light rod) and is intended to have a period of 1.00 seconds. However, the clock runs slow by ten minutes each day. What percentage change should be made in the length of the pendulum?

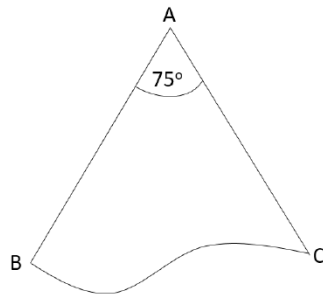
[3]

s) A binary star system is 2140 light years away and consists of two stars like the Sun. The average separation between the stars is 0.00593 light years. Determine the diameter of the telescope needed to resolve them if using a visible wavelength of 550 nm.

[2]

- t) A glass prism of refracting angle  $75.0^\circ$  is shown in **Fig. 4** has a refractive index of  $n = 1.40$ .
- (i) For what range of incident angles will light from air that is incident on face **AB** emerge from face **AC**?
  - (ii) Show your result on a diagram.

[3]



**Figure 4:** Glass prism with an apex angle of  $75^\circ$ .

u) A particle **A**, of mass  $m$  carrying a charge of  $Q$  is suspended by an insulating thread of length  $\ell$ . Another particle **B**, of negligible mass but of positive charge  $+q$  is brought towards **A**, which is repelled. When **B** arrives at the point previously occupied by **A**, the system is in (neutral) equilibrium.

Calculate the work done in terms of  $m, g, \ell, q, Q$ , and  $k$ , where  $k = \frac{1}{4\pi\epsilon_0}$ .

[6]

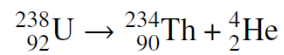
- v) A capacitor of value  $1.0\text{ F}$  discharges through a device whose resistance  $R$  varies linearly with applied potential difference,  $V$ , so that  $R = AV + B$ , where  $A$  and  $B$  are constants. The resistance of the device has a value of  $10.0\ \Omega$  when  $V = 6.0\text{ V}$ , and  $4.0\ \Omega$  when  $V = 0.06\text{ V}$ .

The capacitor is initially charged to a potential of  $6.0\text{ V}$ .

Determine how long it takes for the capacitor to discharge to  $1\%$  of this initial value.

**[6]**

w) The  ${}^{238}_{92}\text{U}$  decays according to



Determine the kinetic energy of the emitted  $\alpha$ -particle in MeV.

$$c = 2.998 \times 10^8 \text{ m s}^{-1}$$

Mass of the  ${}^{238}_{92}\text{U}$  nucleus is  $3.85395 \times 10^{-25} \text{ kg}$

Mass of the  ${}^{234}_{90}\text{Th}$  nucleus is  $3.78737 \times 10^{-25} \text{ kg}$

Mass of the  $\alpha$ -particle is  $6.64807 \times 10^{-27} \text{ kg}$

[5]